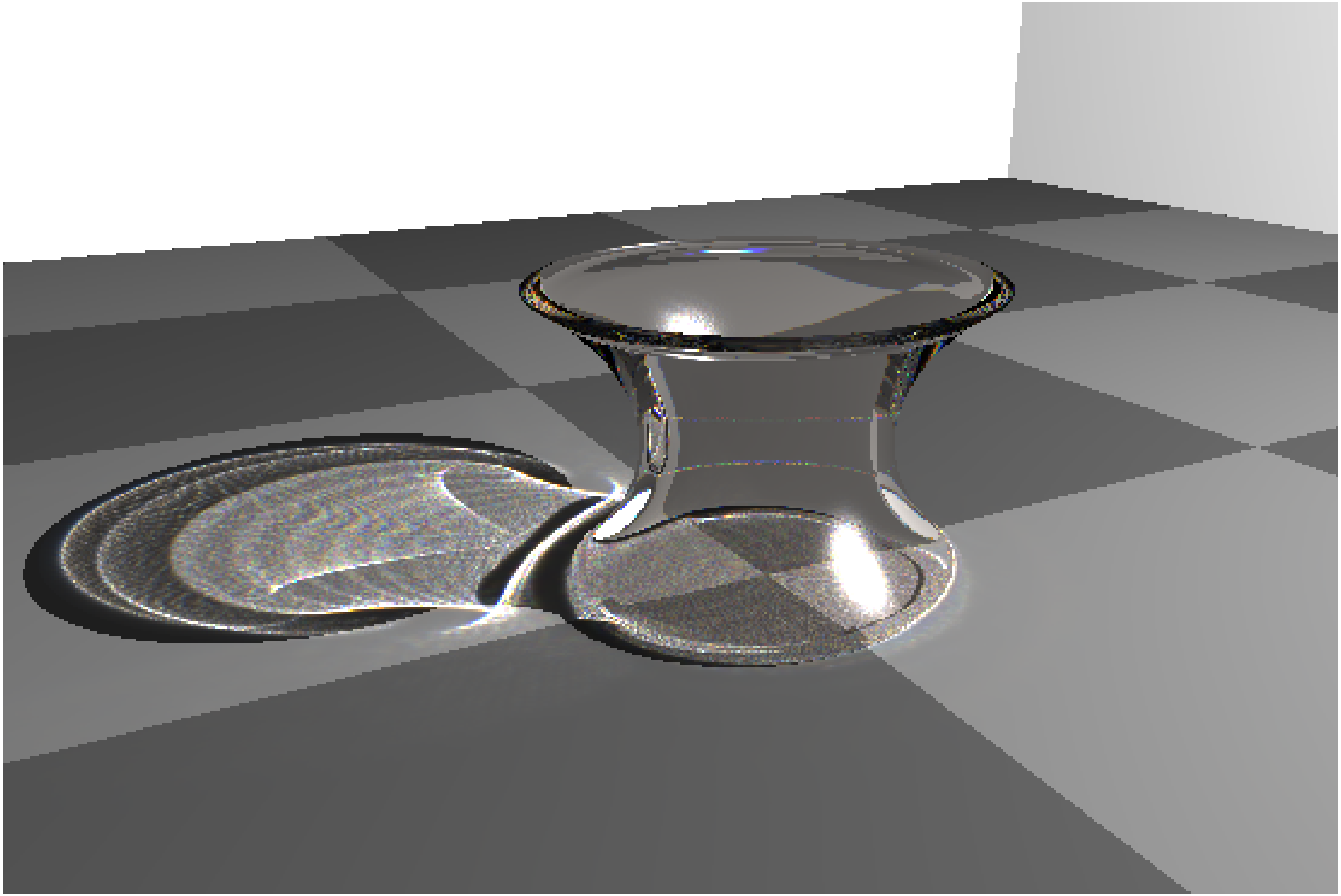

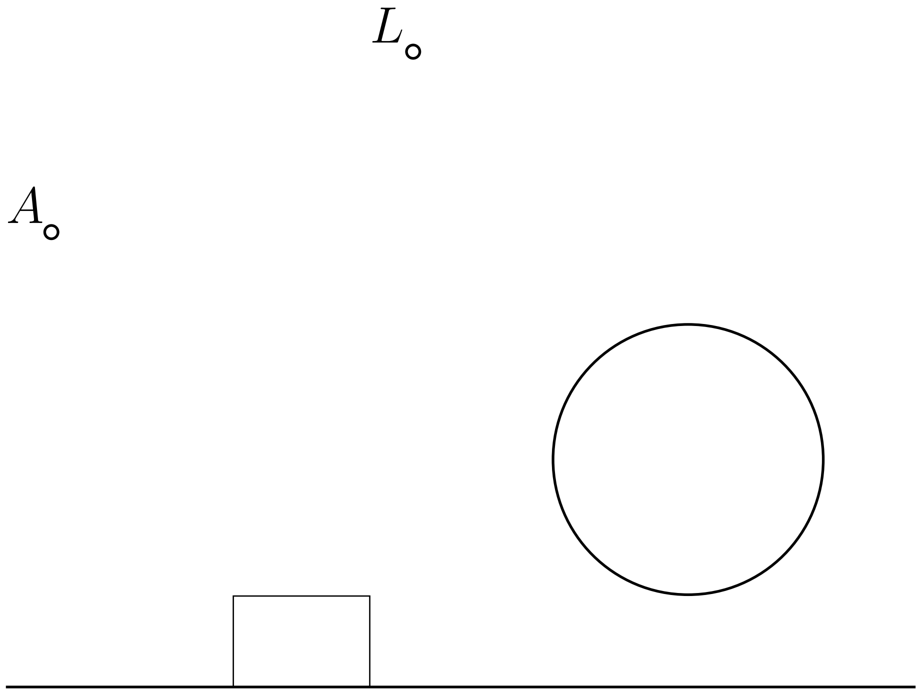

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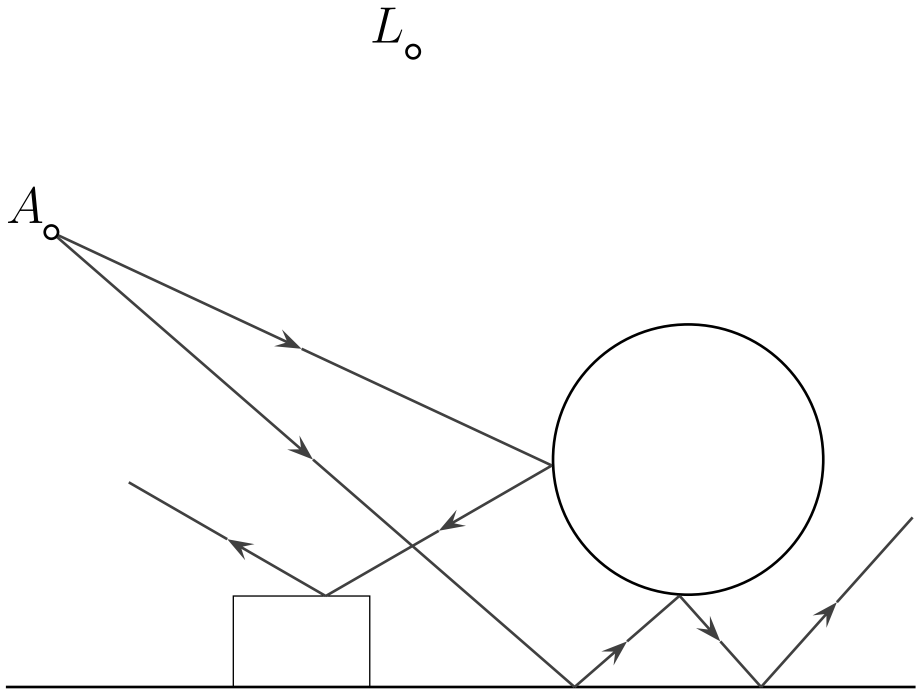
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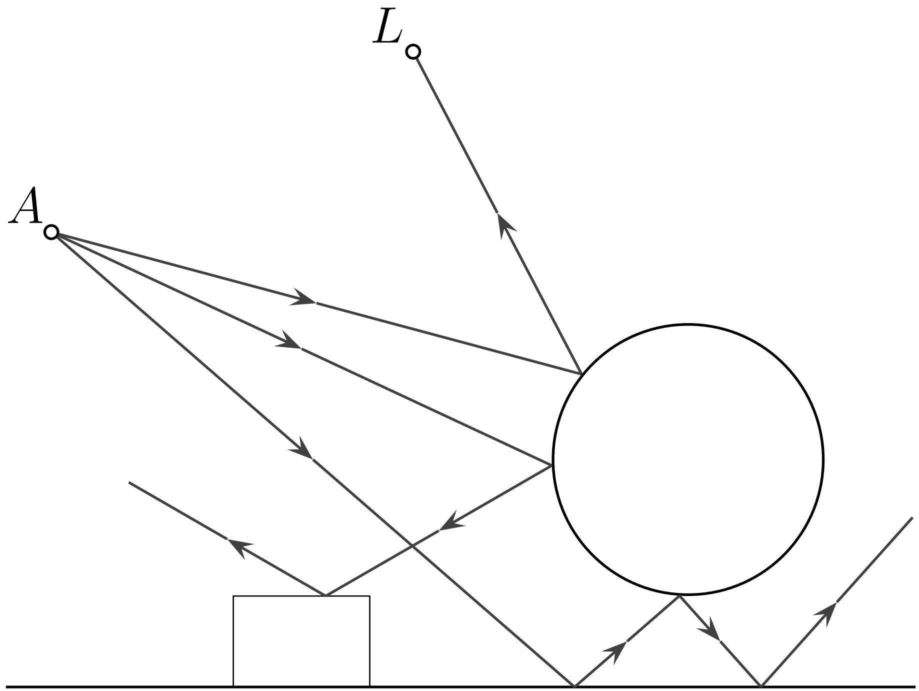
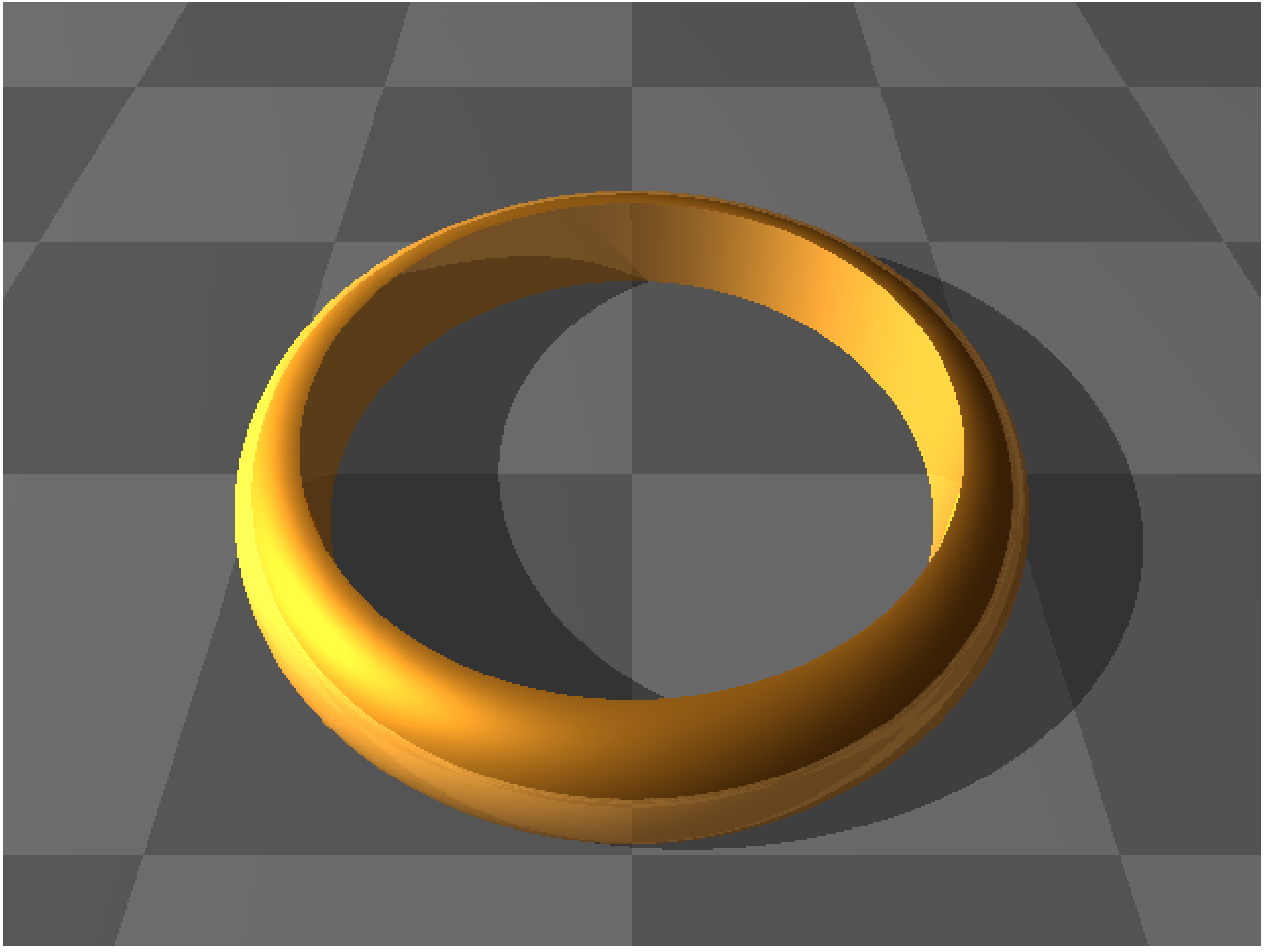

L
A

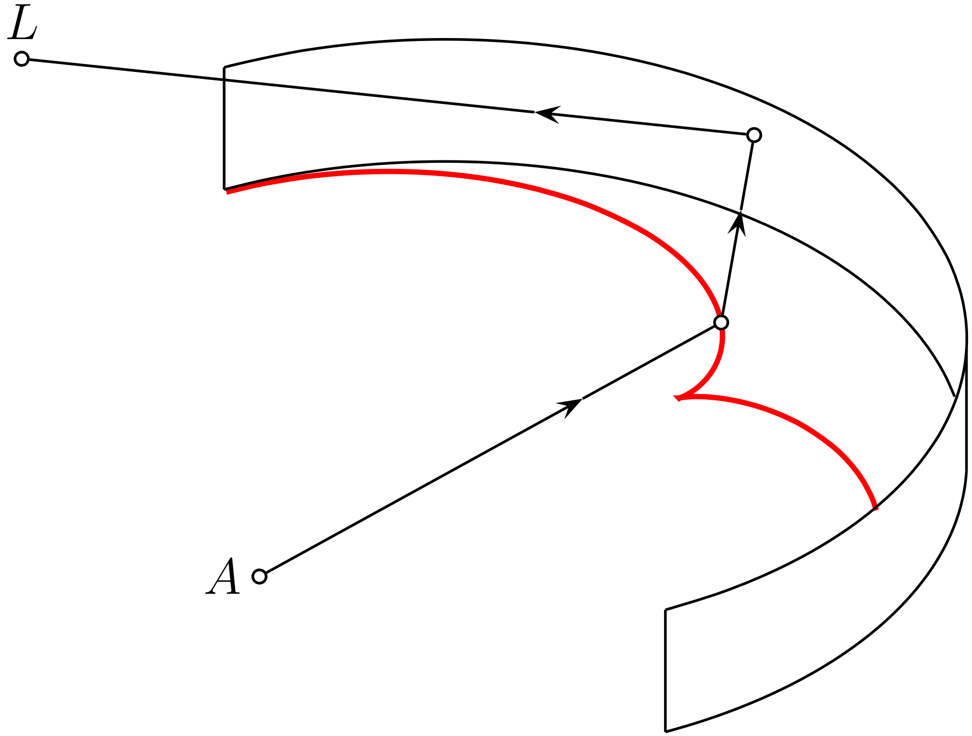
```
#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}
```

```
#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}

object{ring texture{T_Gold_1A}
       translate<0,1,0>
      }
```

```
camera {location <0,20,-16> look_at <0,0,0>}

light_source {<-120,80,-20> color White*3}





#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}

object{ring texture{T_Gold_1A}
       translate<0,1,0>
      }
```

```
camera {location <0,20,-16> look_at <0,0,0>}

light_source {<-120,80,-20> color White*3}

plane{<0,1,0>,0
      pigment{
        checker color Gray40 color Gray50 scale <10,10,10>}
      finish{ambient 0.5 diffuse 0.2}}

#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}

object{ring texture{T_Gold_1A}
       translate<0,1,0>
      }
```

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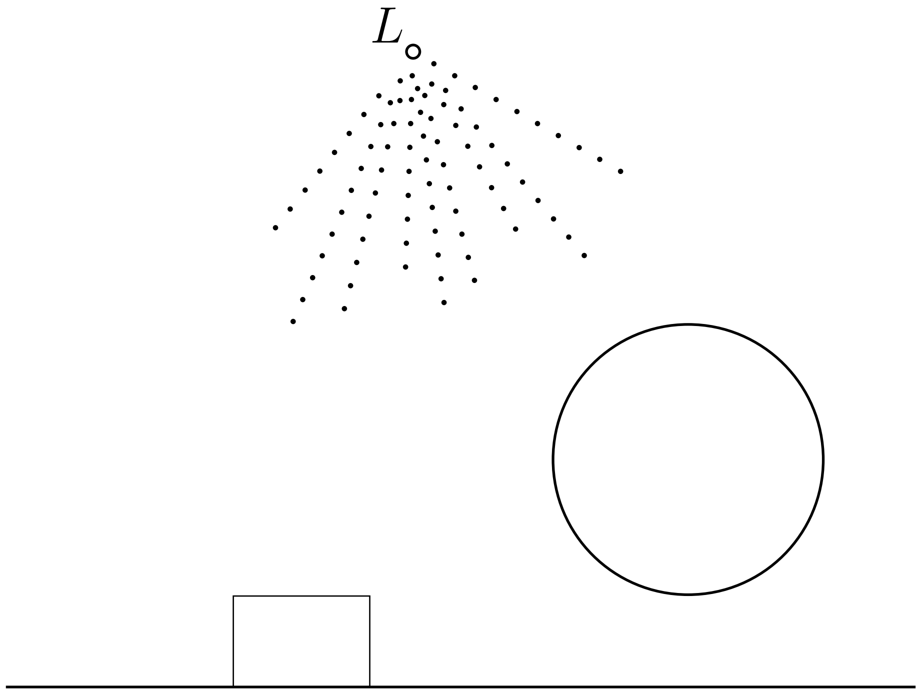
A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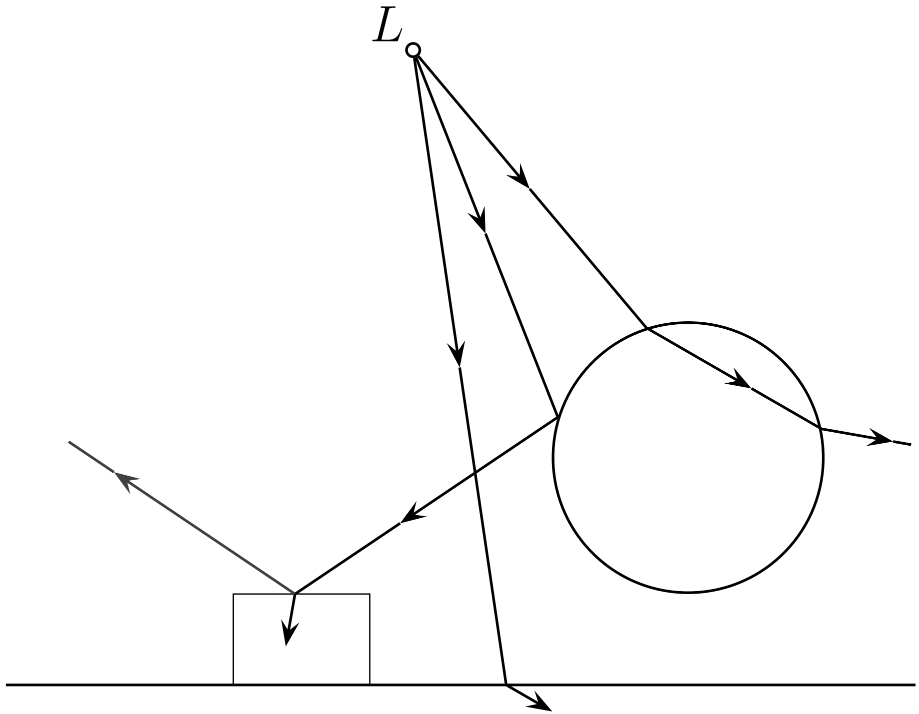
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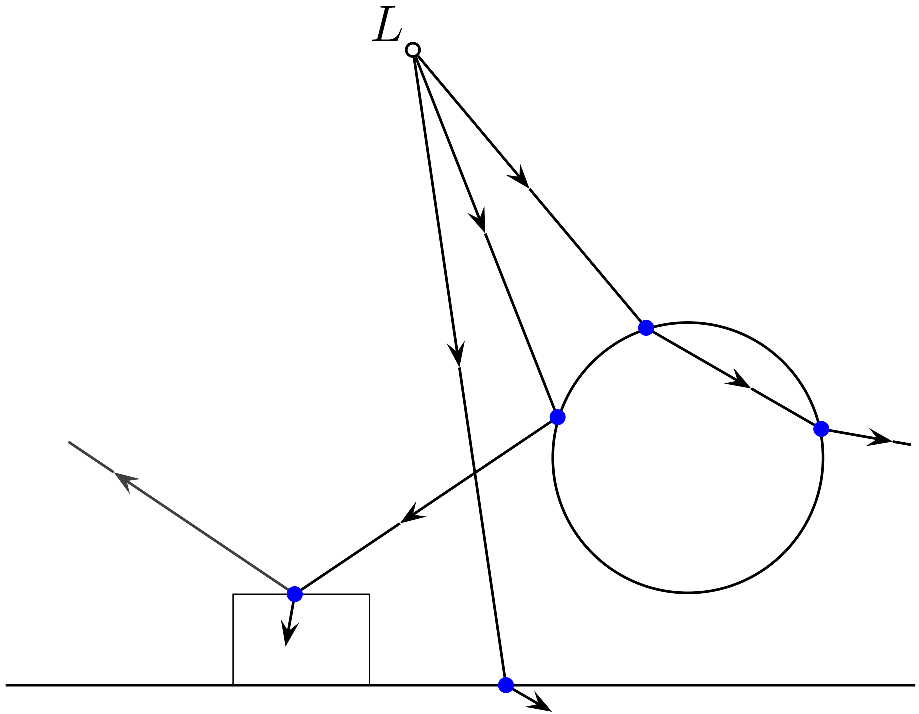
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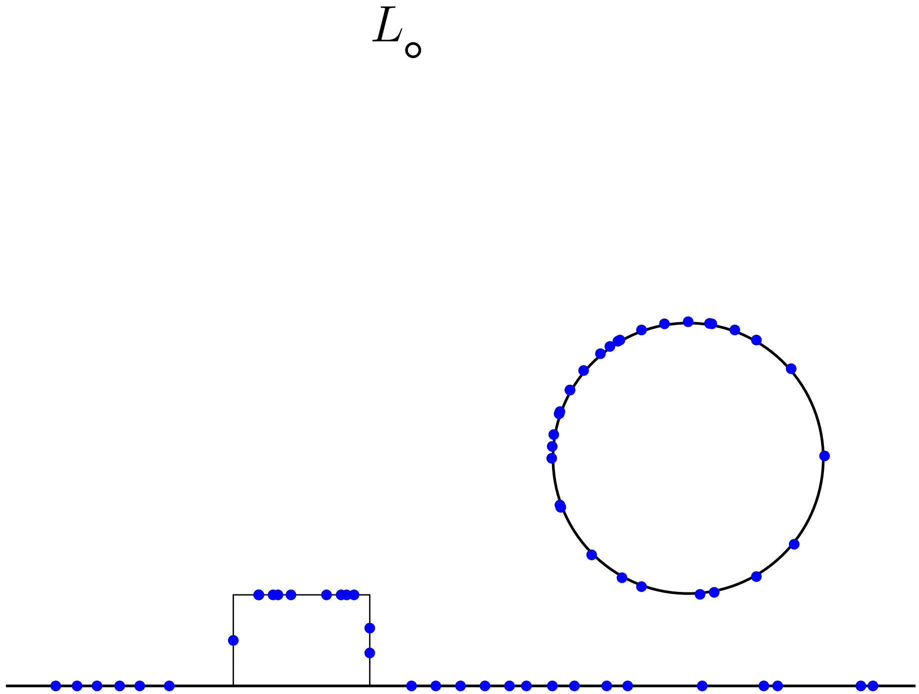
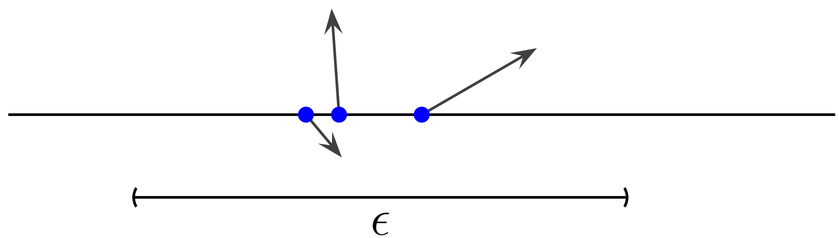

$L_\circ$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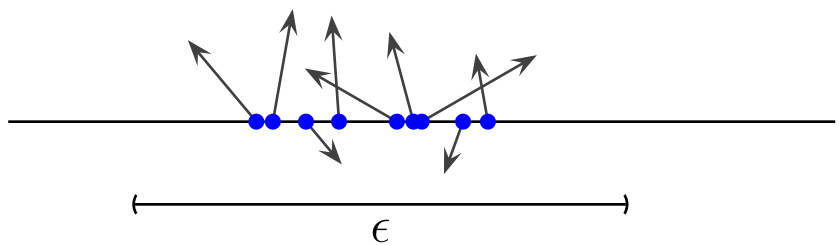
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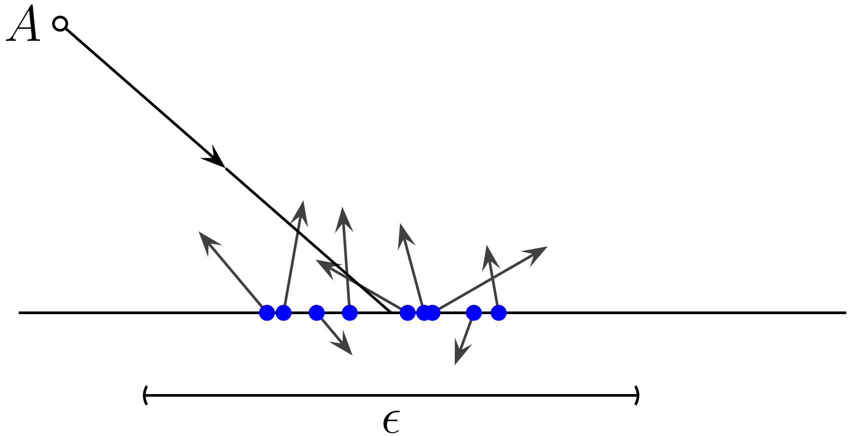
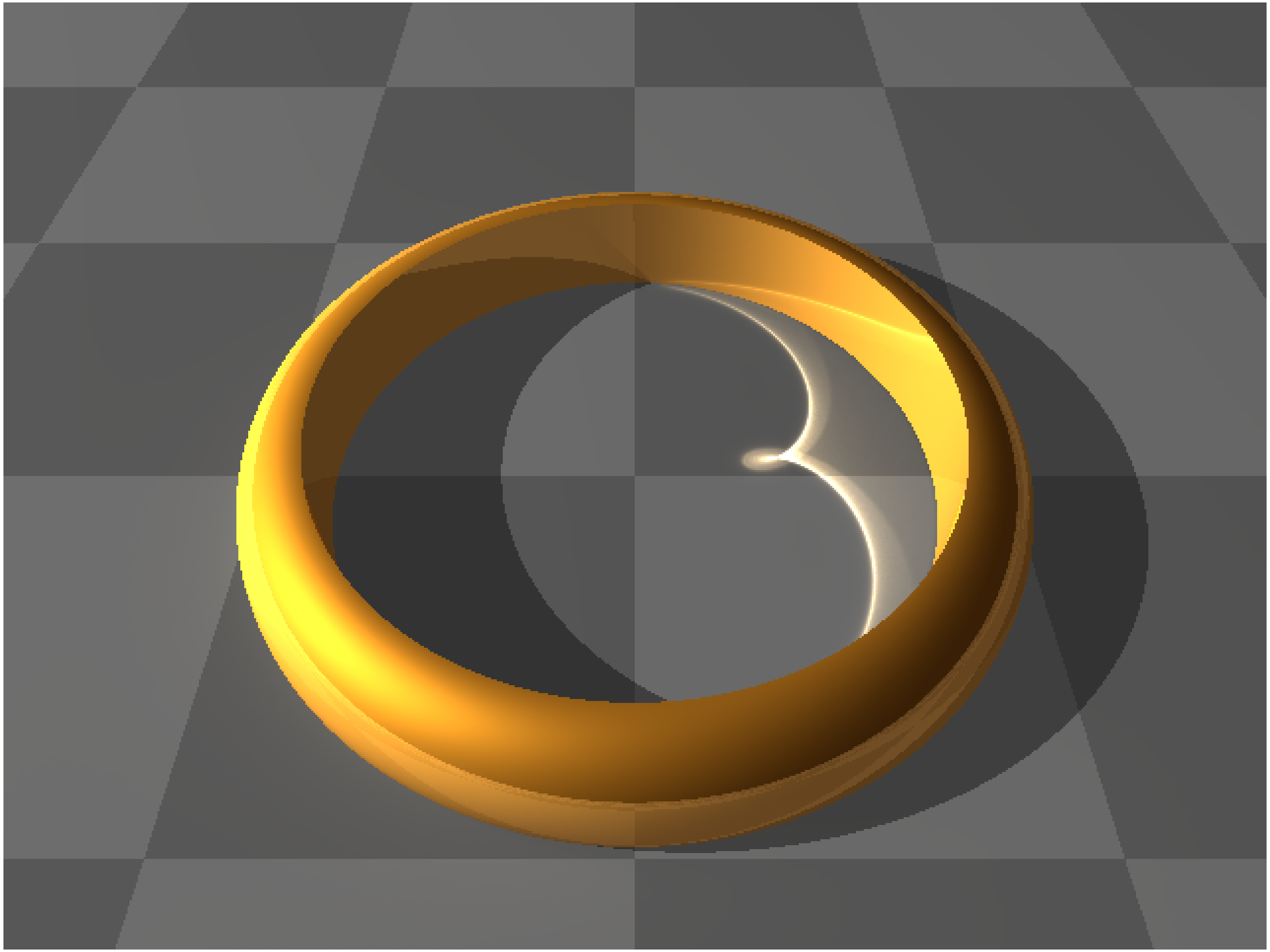
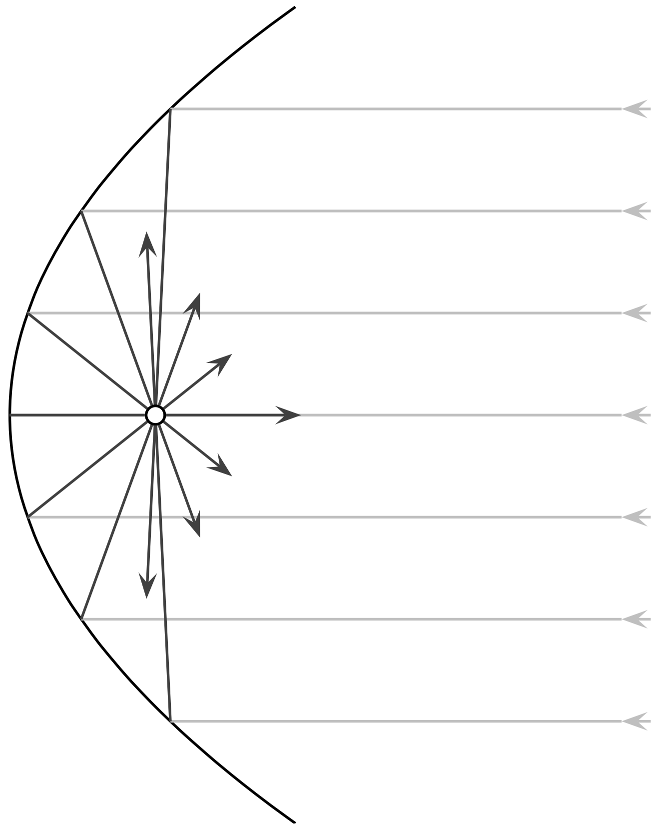
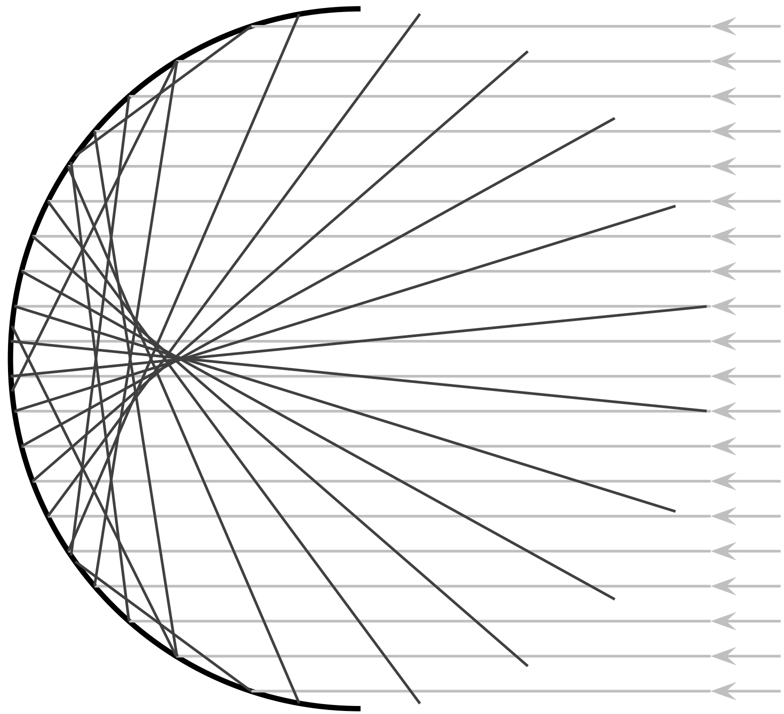

A
ϵ

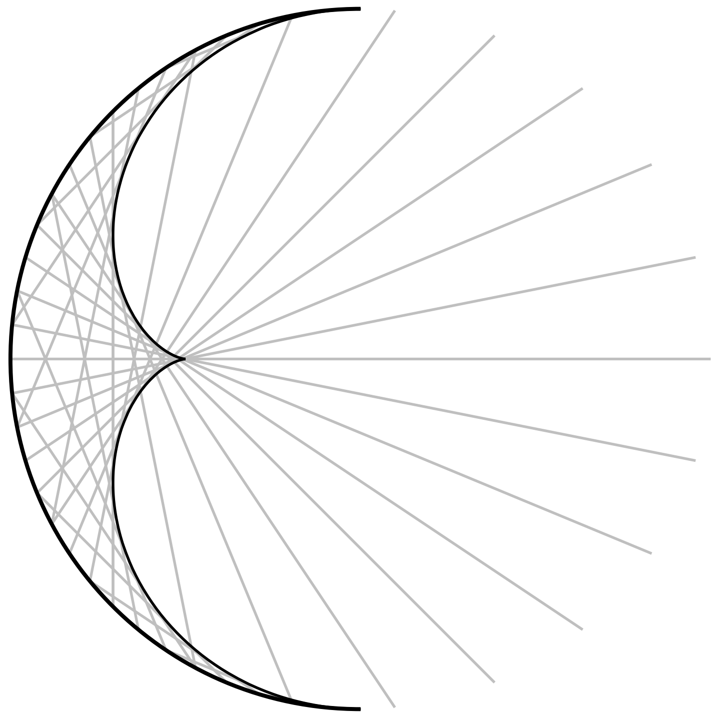
```
camera {location <0,20,-16> look_at <0,0,0>}

light_source {<-120,80,-20> color White*3}

plane{<0,1,0>,0
      pigment{
        checker color Gray40 color Gray50 scale <10,10,10>}
      finish{ambient 0.5 diffuse 0.2}}

#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}

object{ring texture{T_Gold_1A}
       translate<0,1,0>

       }
```

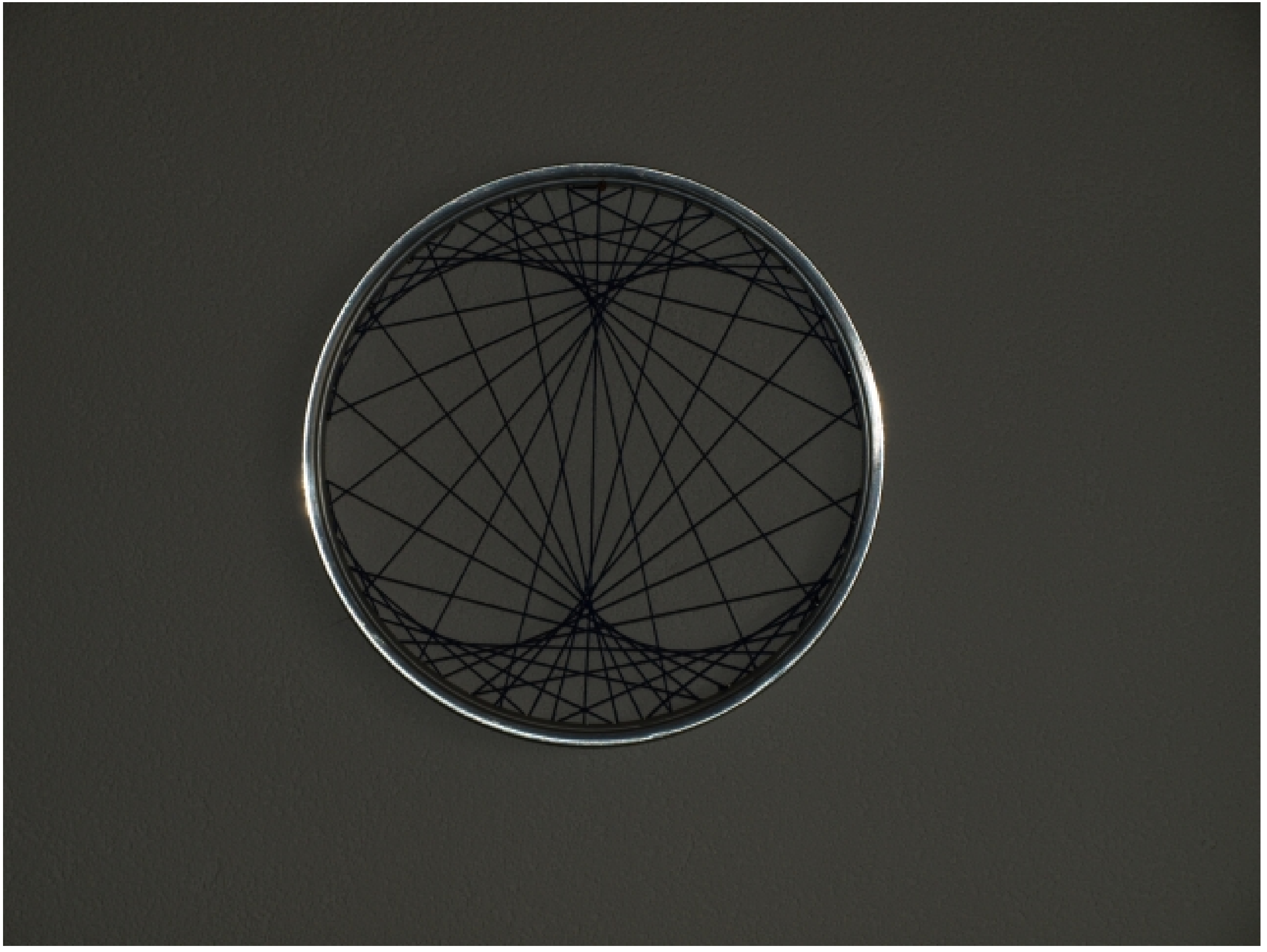
```
global_settings{ photons{count 5000000} }

camera {location <0,20,-16> look_at <0,0,0>}

light_source {<-120,80,-20> color White*3}

plane{<0,1,0>,0
      pigment{
        checker color Gray40 color Gray50 scale <10,10,10>}
      finish{ambient 0.5 diffuse 0.2}}

#declare ring=difference{
            torus{8,2} cylinder{<0,-2,0>,<0,4,0>,8}}

object{ring texture{T_Gold_1A}
       translate<0,1,0>
       photons {target refraction on reflection on}
       }
```

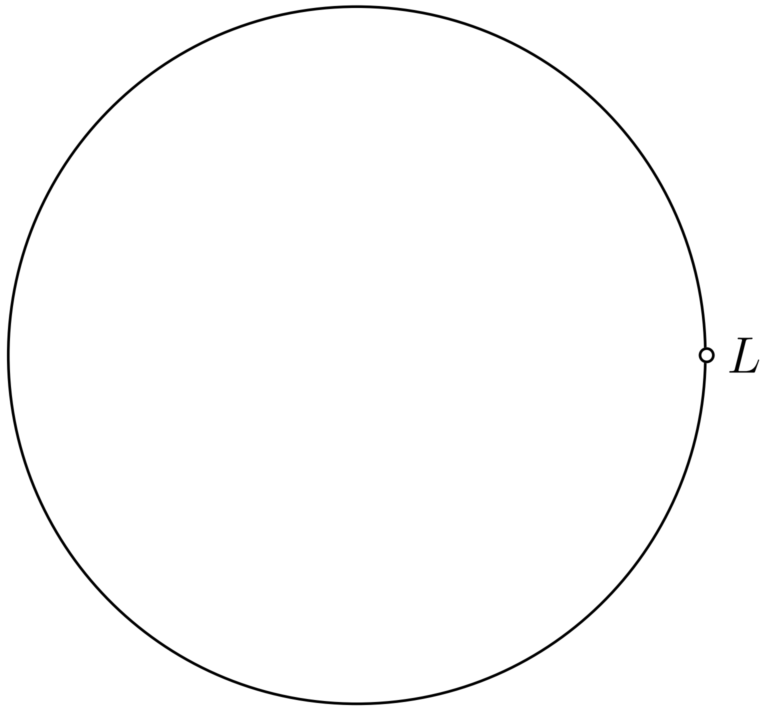








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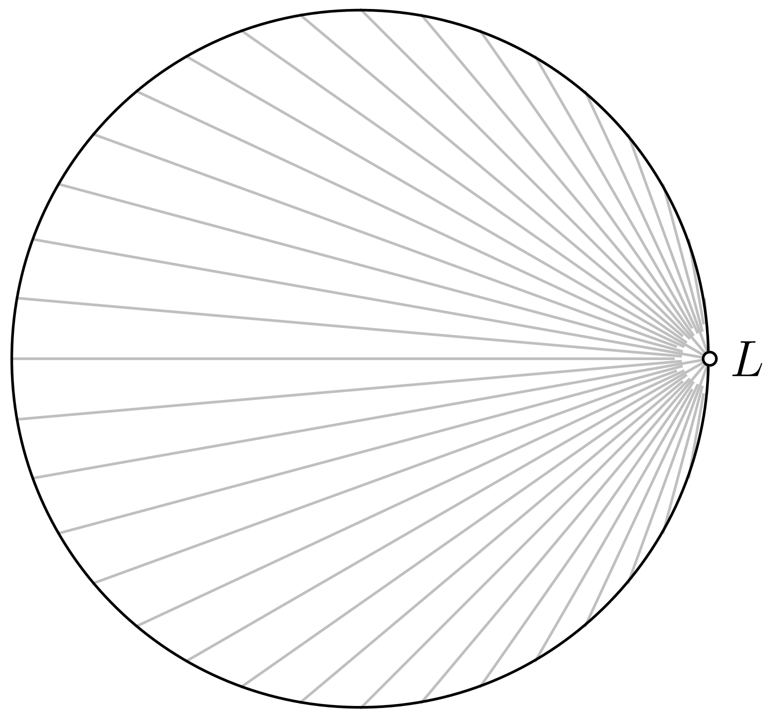
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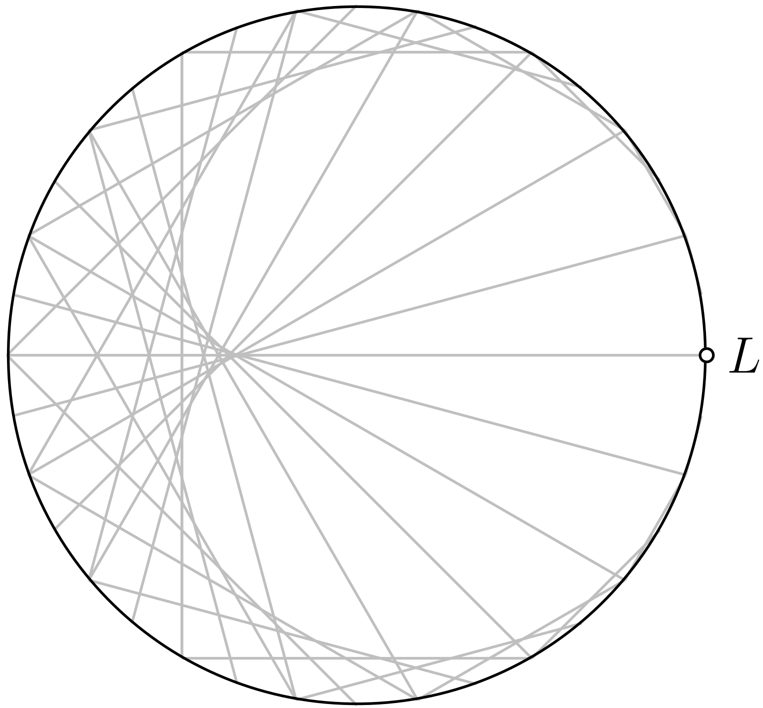
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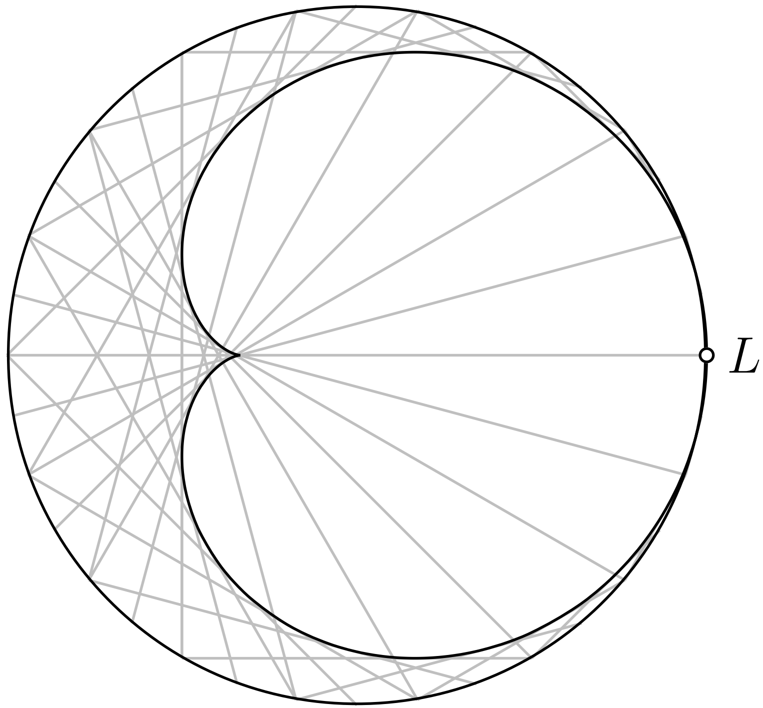
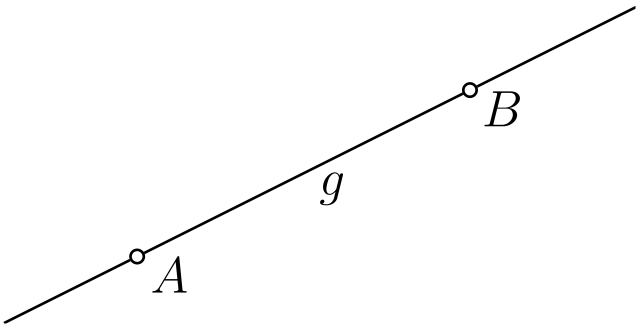
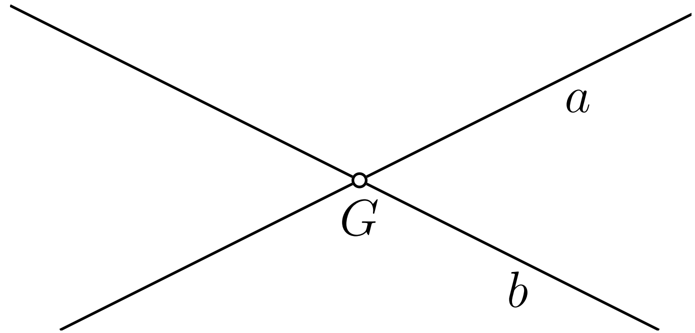
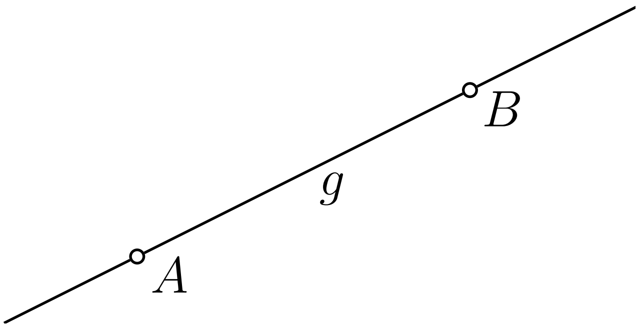

L



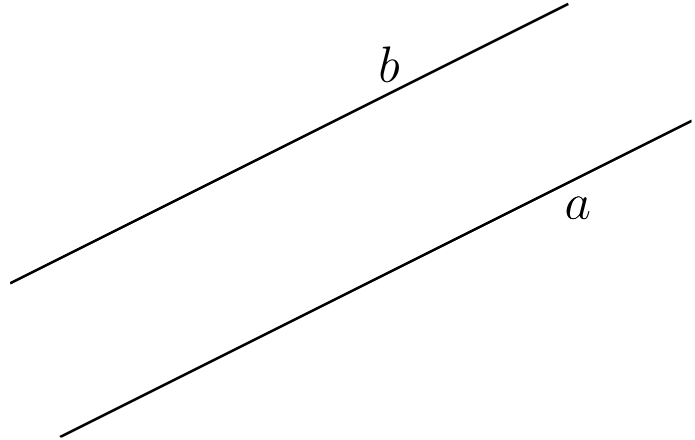
Zwei Punkte haben eine Gerade gemeinsam



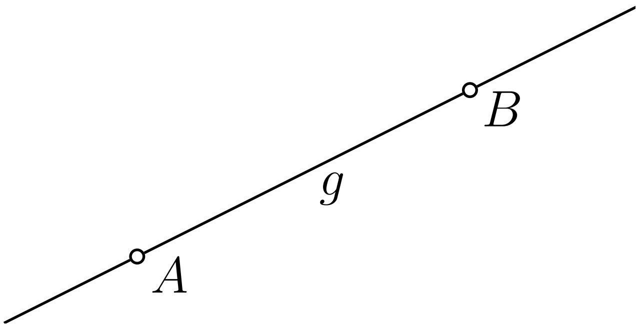
Zwei Geraden haben einen Punkt gemeinsam

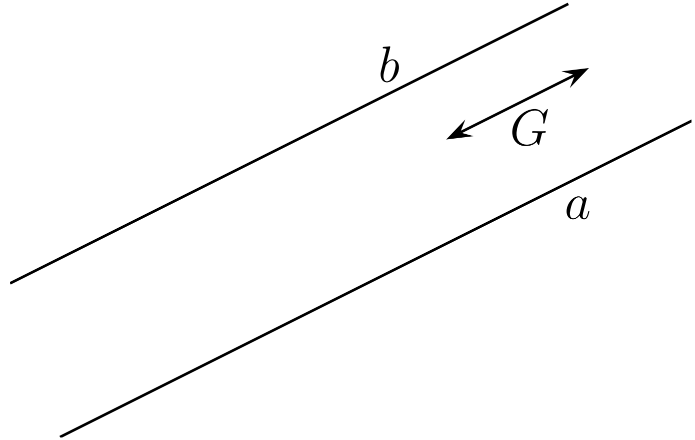


Zwei Punkte haben eine Gerade gemeinsam

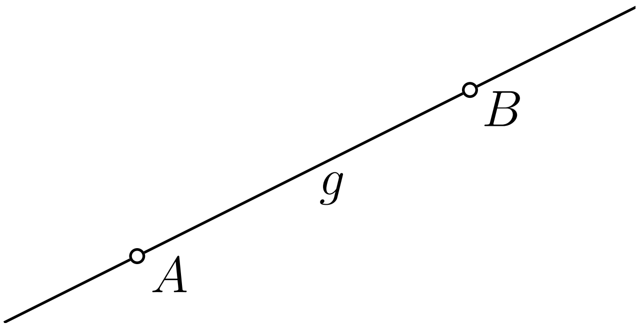


~~Zwei Geraden haben einen Punkt gemeinsam~~

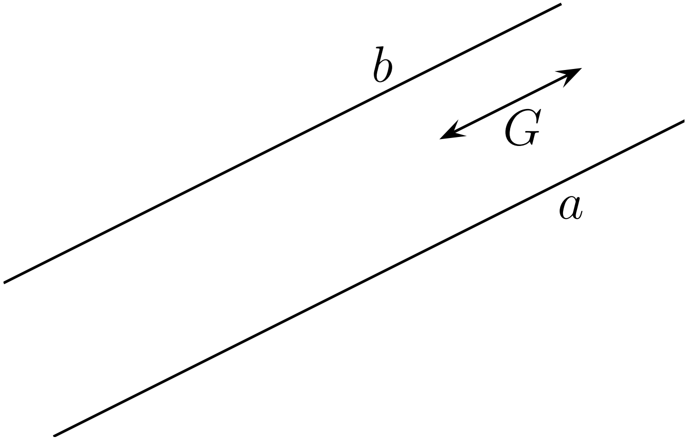




Zwei Punkte haben eine
Gerade gemeinsam



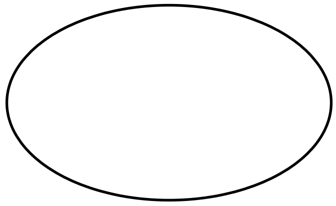
Zwei Punkte haben eine Gerade gemeinsam



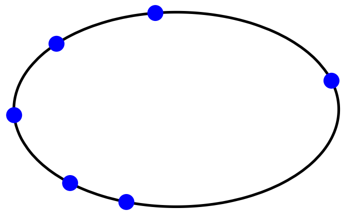
Zwei Geraden haben einen Punkt gemeinsam

Satz von Pascal (1640)

Satz von Pascal (1640)

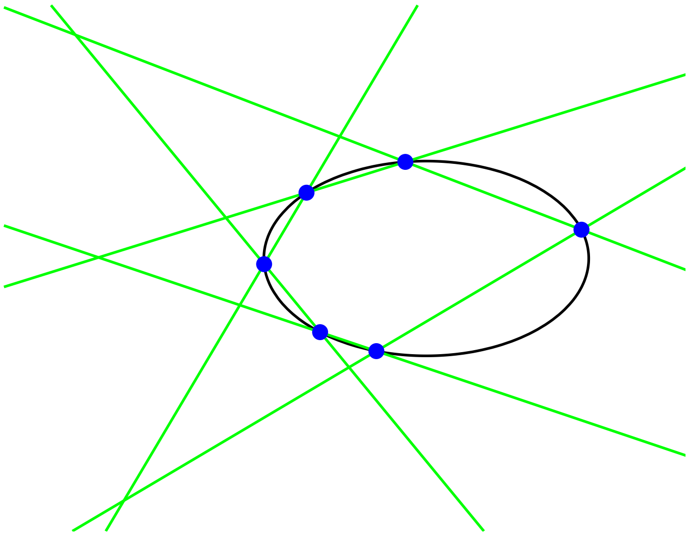


Satz von PASCAL (1640)



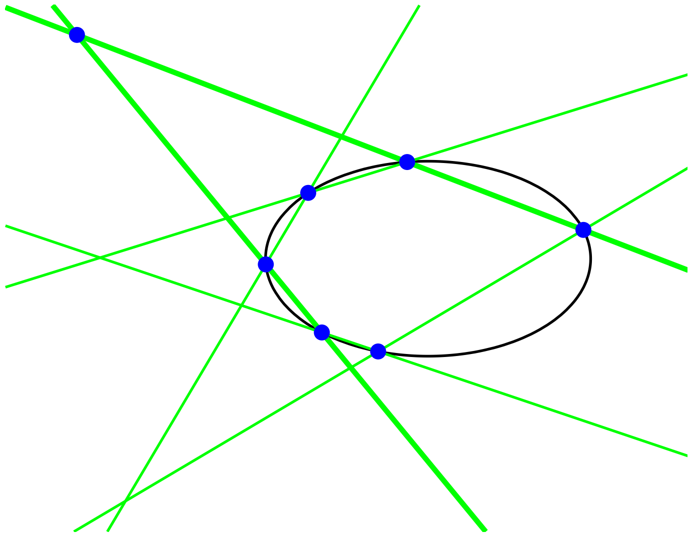
6 Kurvenpunkte

Satz von PASCAL (1640)



6 Kurvenpunkte, Sechsseit

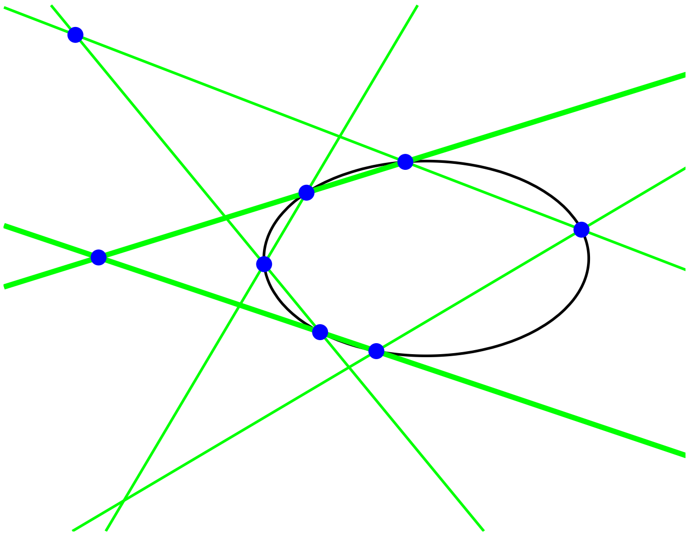
Satz von Pascal (1640)



6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

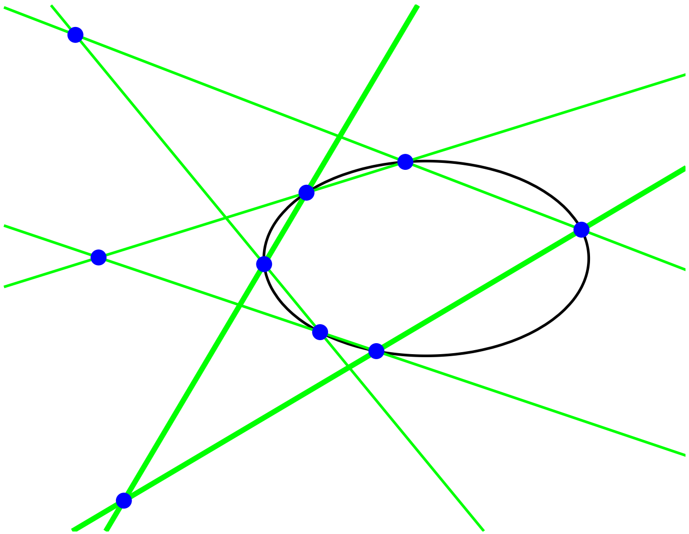
Satz von Pascal (1640)



6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

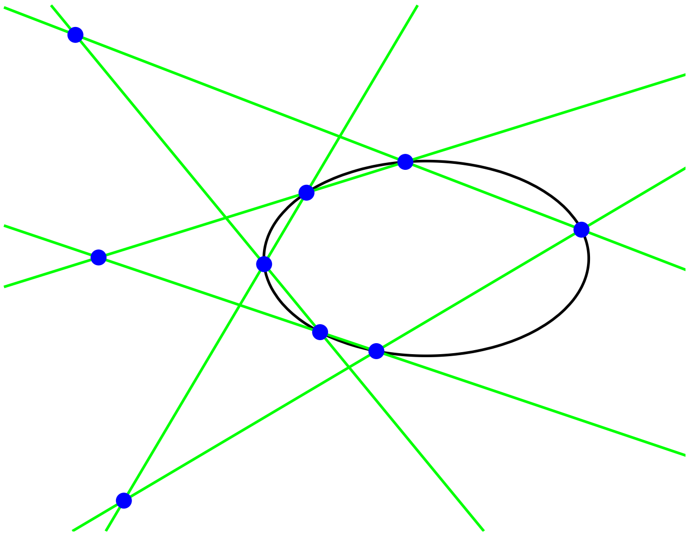
Satz von Pascal (1640)



6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

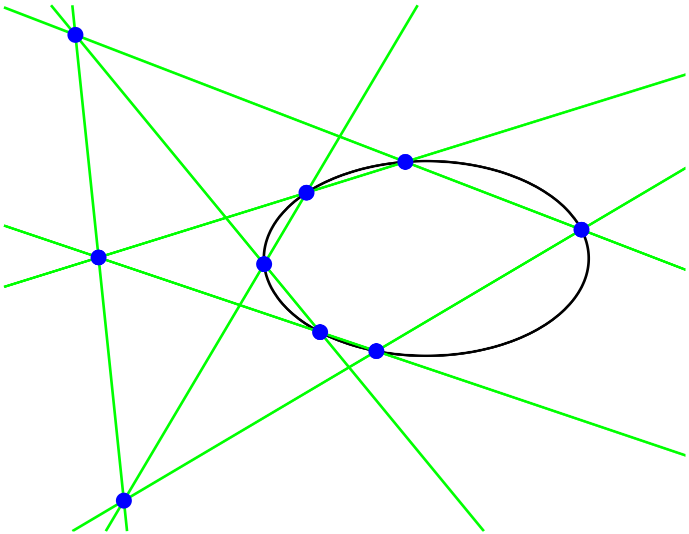
Satz von PASCAL (1640)



6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

Satz von PASCAL (1640)



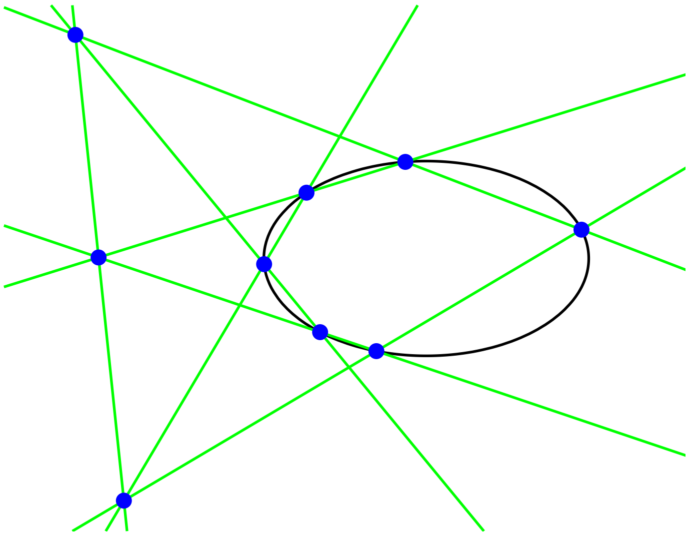
6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

Satz von PASCAL (1640)

Satz von BRIANCHON (1806)



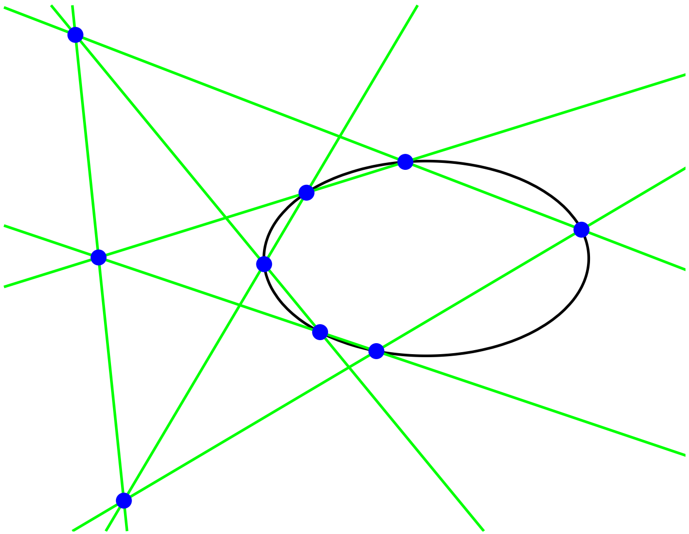
6 Kurvenpunkte, Sechsseit

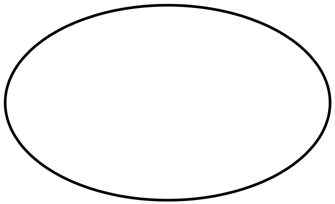
je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

Satz von Pascal (1640)

Satz von Brianchon (1806)





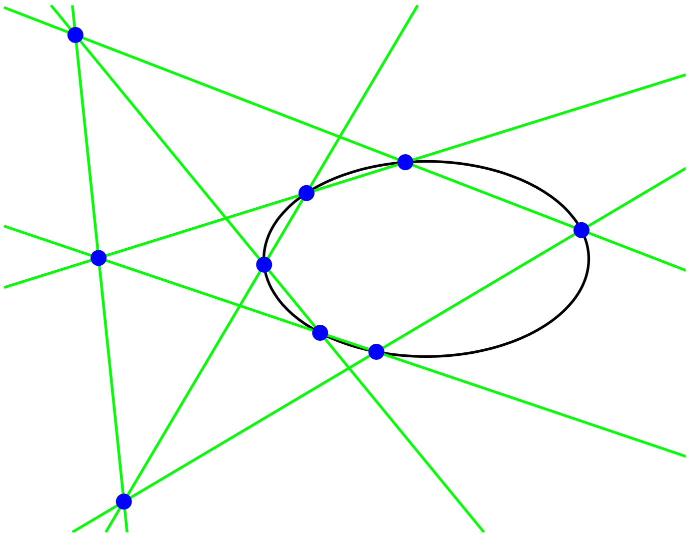
6 Kurvenpunkte, Sechsseit

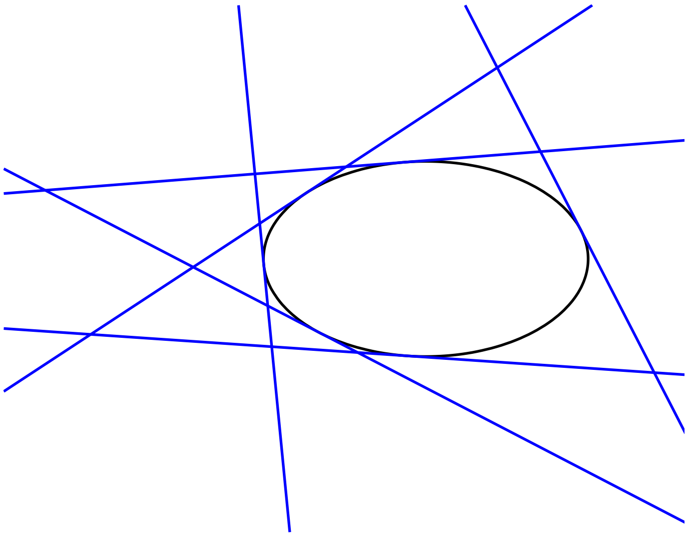
je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

Satz von Pascal (1640)

Satz von Brianchon (1806)





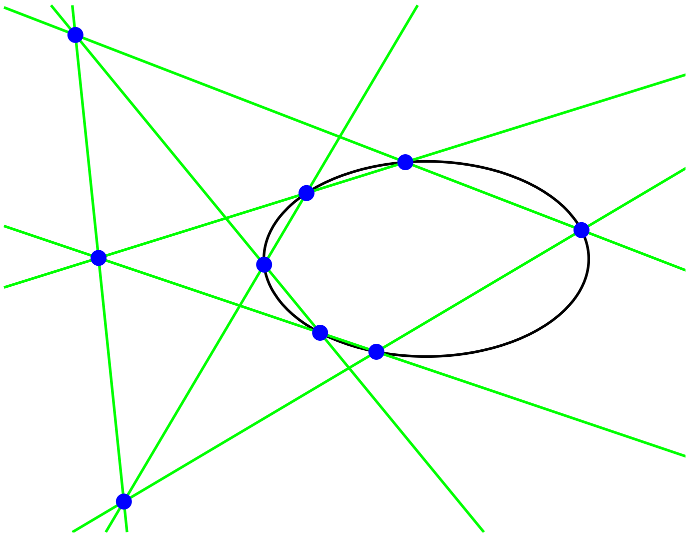
6 Kurvenpunkte, Sechsseit

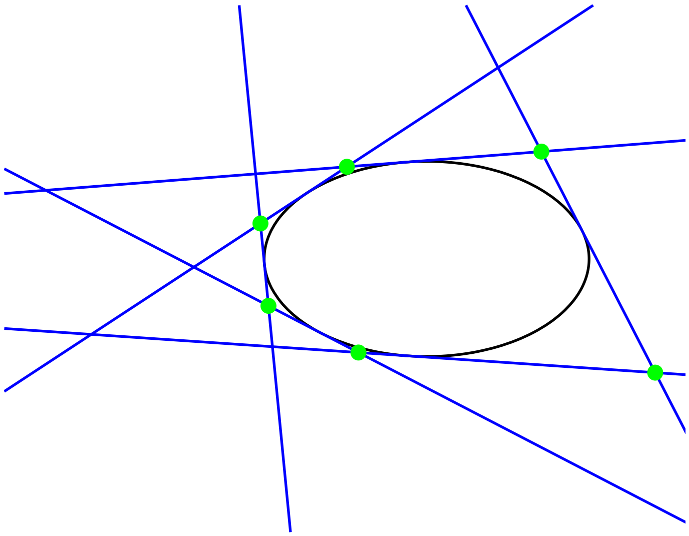
je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

6 Tangenten

Satz von Pascal (1640) Satz von Brianchon (1806)





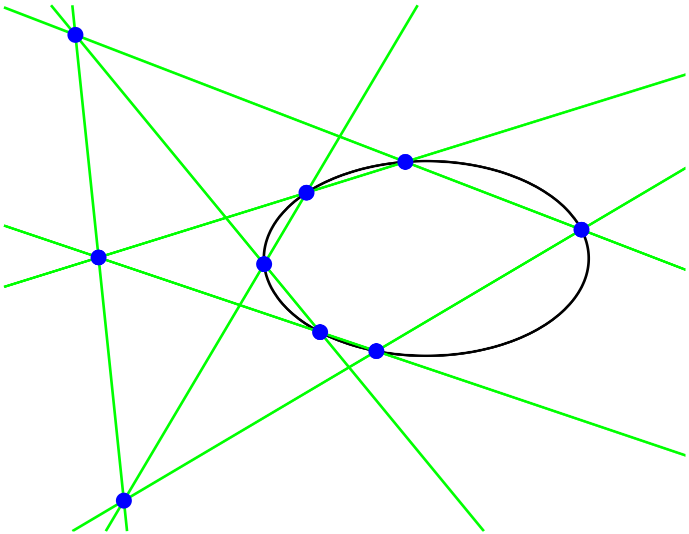
6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

6 Tangenten, Sechseck

Satz von PASCAL (1640)

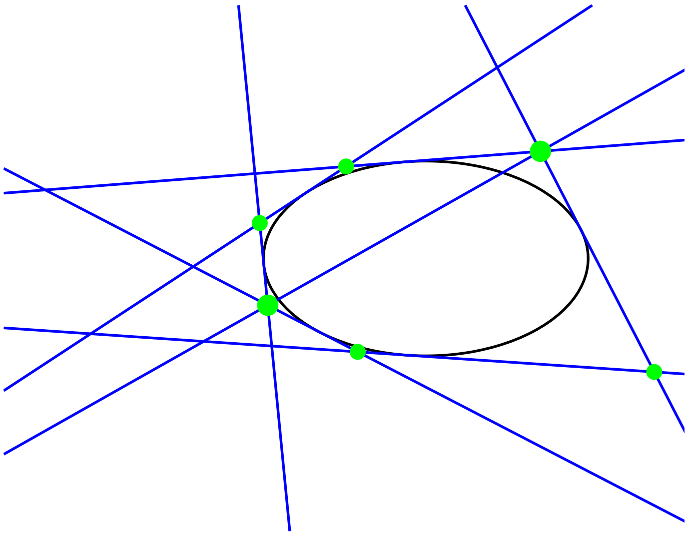


6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

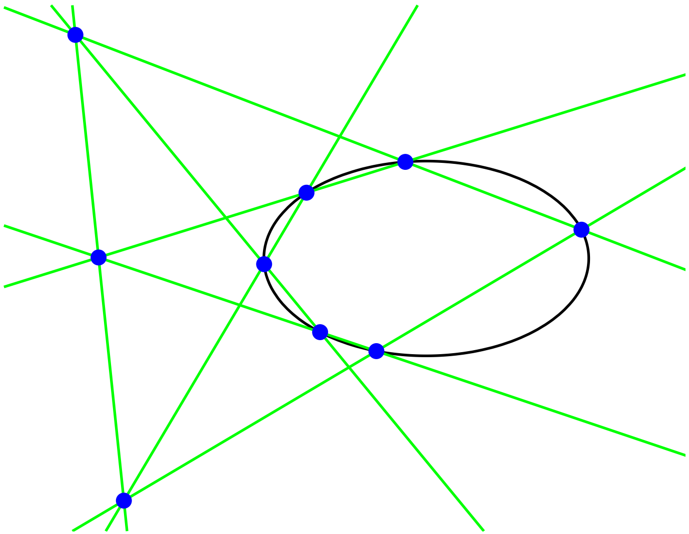
Satz von BRIANCHON (1806)



6 Tangenten, Sechseck

je zwei Gegenecken verbinden

Satz von Pascal (1640)

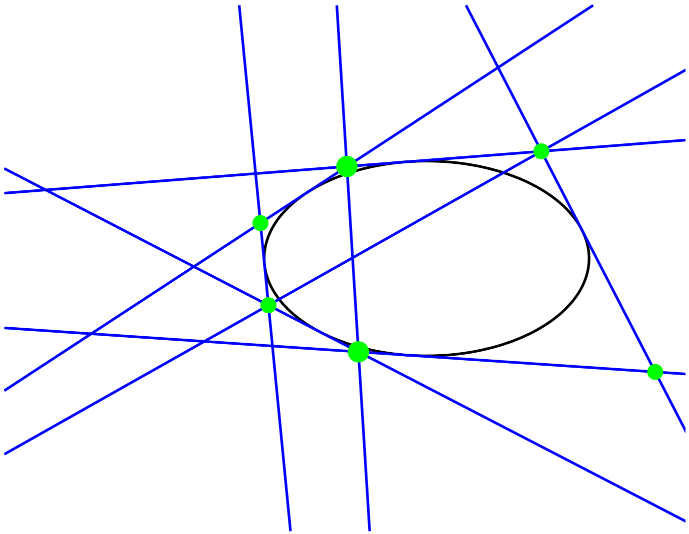


6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

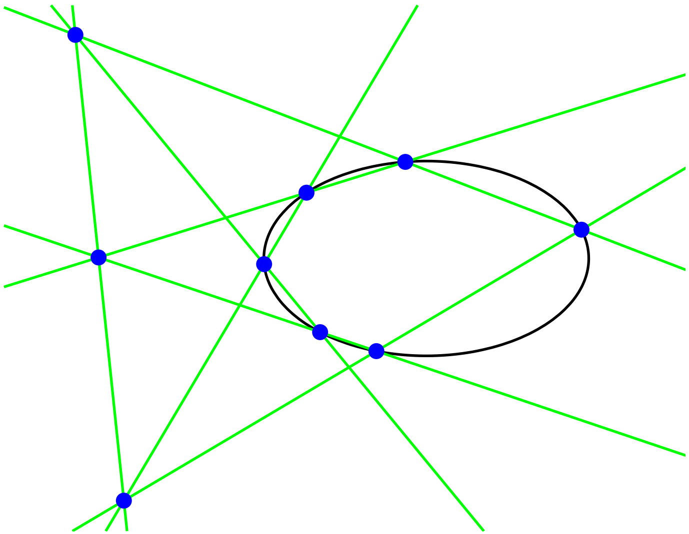
Satz von Brianchon (1806)



6 Tangenten, Sechseck

je zwei Gegenecken verbinden

Satz von PASCAL (1640)

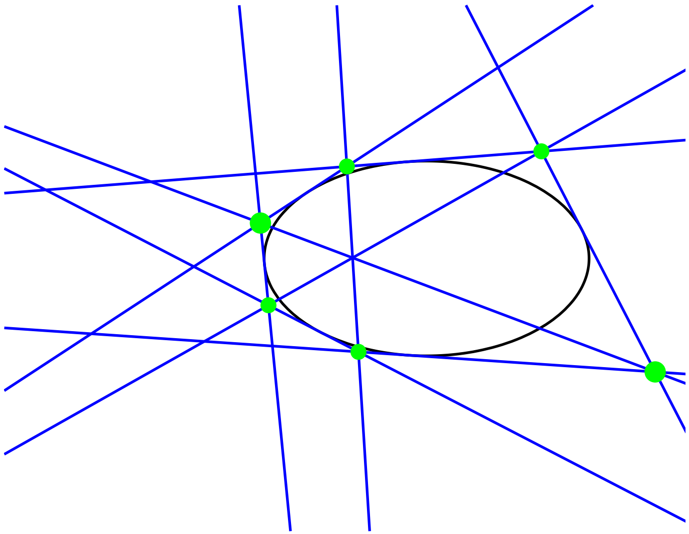


6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

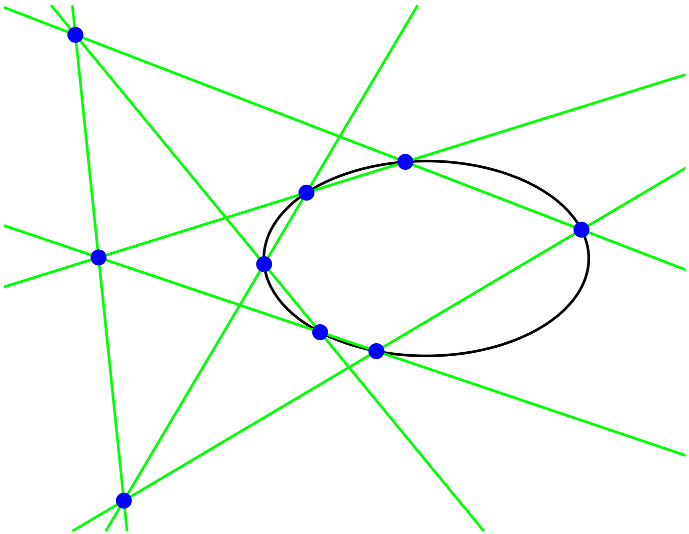
Satz von BRIANCHON (1806)



6 Tangenten, Sechseck

je zwei Gegenecken verbinden

Satz von PASCAL (1640)

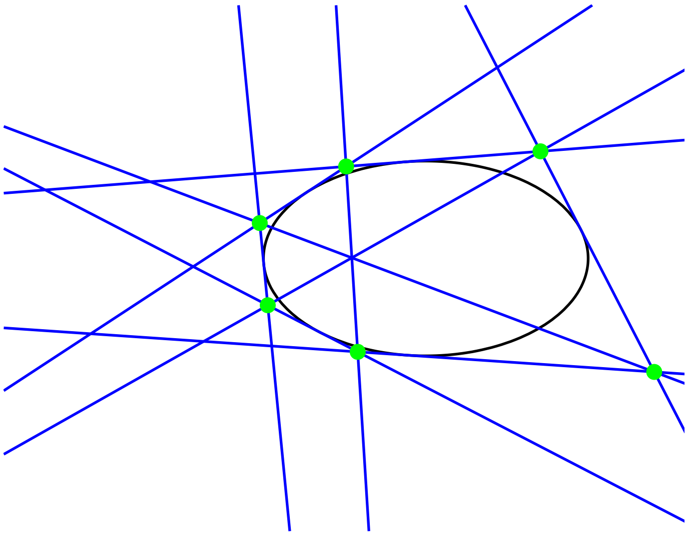


6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

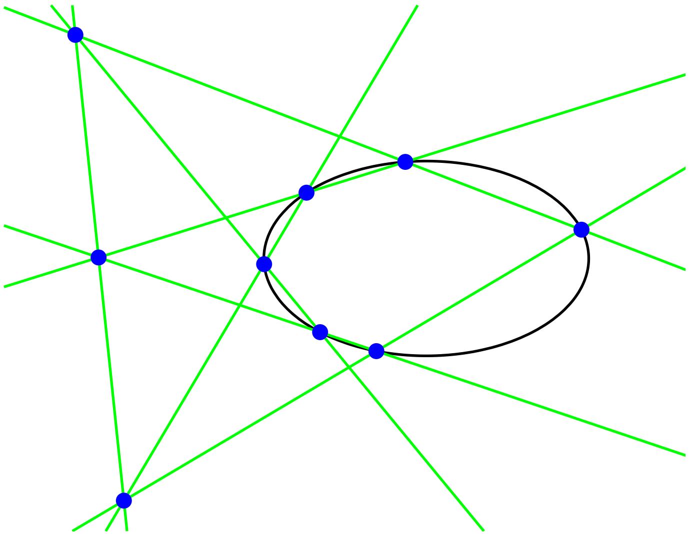
Satz von BRIANCHON (1806)



6 Tangenten, Sechseck

je zwei Gegenecken verbinden

Satz von Pascal (1640)

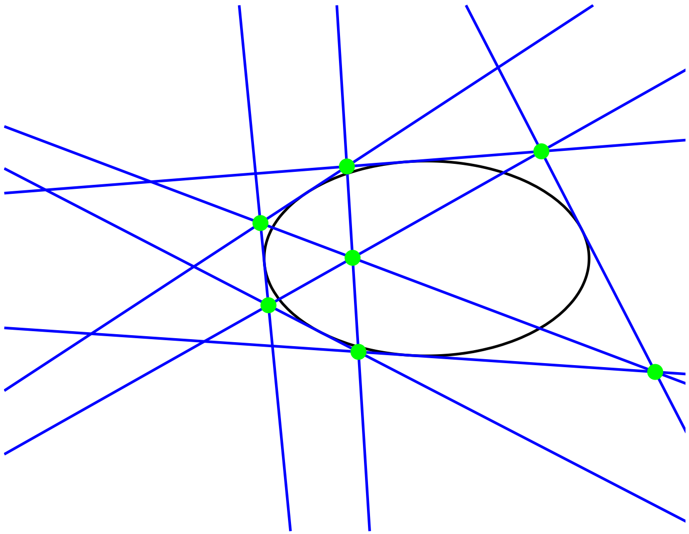


6 Kurvenpunkte, Sechsseit

je zwei Gegenseiten schneiden

die 3 Punkte sind kollinear

Satz von Brianchon (1806)



6 Tangenten, Sechseck

je zwei Gegenecken verbinden

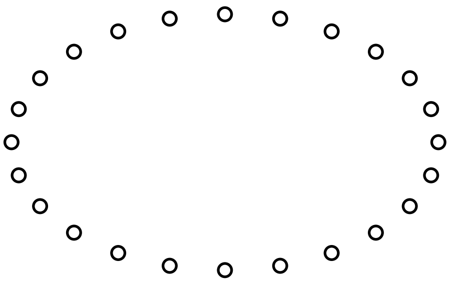
die 3 Geraden sind kopunktal

Wörterbuch

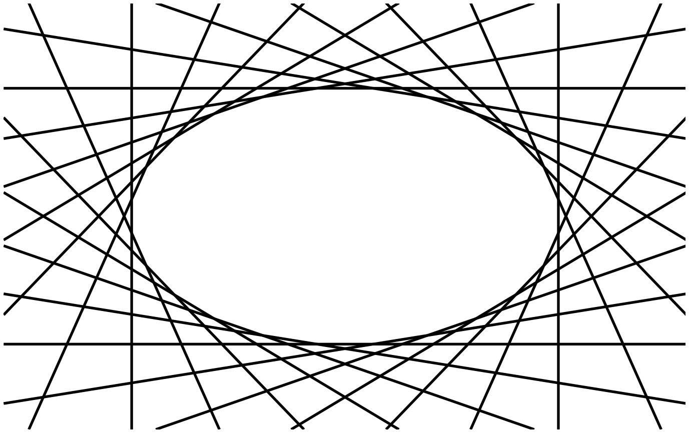
Punkt	Gerade
liegt auf	geht durch
Gerade durch zwei Punkte	Schnittpunkt von zwei Geraden
kollinear	kopunktal
Dreieck	Dreiseit
Pol	Polare

Wörterbuch

Ortskurve



Hüllkurve



Das Dualitätsprinzip in der projektiven Geometrie:

Die duale Übersetzung eines wahren Satzes ist wieder ein wahrer Satz.

projektive Koordinaten eines Punktes

$$(p_1, p_2, p_3)$$

projektive Koordinaten eines Punktes

$$(p_1, p_2, p_3)$$

$$(p_1, p_2, p_3) \equiv (r_1, r_2, r_3) \iff (p_1, p_2, p_3) = \lambda(r_1, r_2, r_3), \quad \lambda \in \mathbb{R} \backslash \{0\}$$

projektive Koordinaten eines Punktes

$$(p_1, p_2, p_3)$$

$$(p_1, p_2, p_3) \equiv (r_1, r_2, r_3) \iff (p_1, p_2, p_3) = \lambda(r_1, r_2, r_3), \quad \lambda \in \mathbb{R}\backslash\{0\}$$

$p_3 = 0$: uneigentlicher Punkt (Fernpunkt)

$p_3 \neq 0$: eigentlicher Punkt

Gleichung einer Geraden

$$g_1x + g_2y + g_3z = 0$$

Gleichung einer Geraden

$$g_1 x + g_2 y + g_3 z = 0$$

$$(g_1, g_2, g_3) \equiv (f_1, f_2, f_3) \iff (g_1, g_2, g_3) = \lambda(f_1, f_2, f_3), \quad \lambda \in \mathbb{R}\backslash\{0\}$$

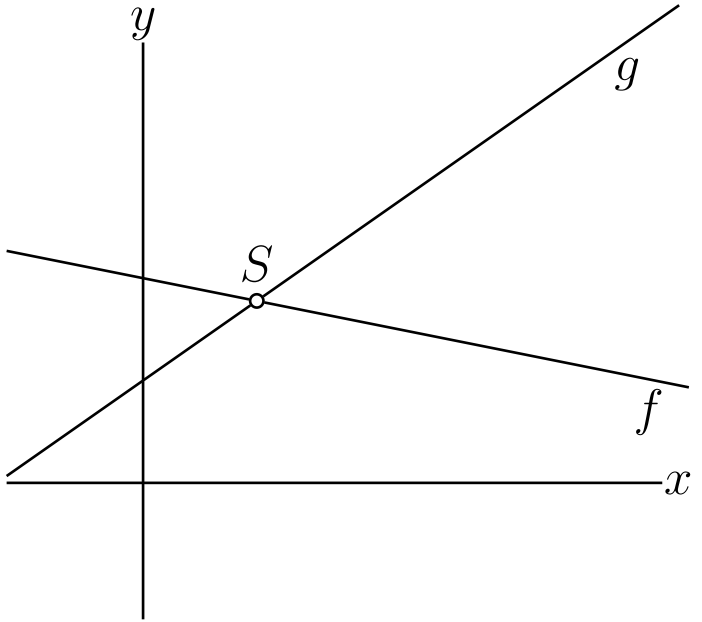
Gleichung einer Geraden

$$g_1 x + g_2 y + g_3 z = 0$$

$$(g_1, g_2, g_3) \equiv (f_1, f_2, f_3) \iff (g_1, g_2, g_3) = \lambda(f_1, f_2, f_3), \quad \lambda \in \mathbb{R}\backslash\{0\}$$

$g_1 = g_2 = 0$: uneigentliche Gerade (Ferngerade $z = 0$)

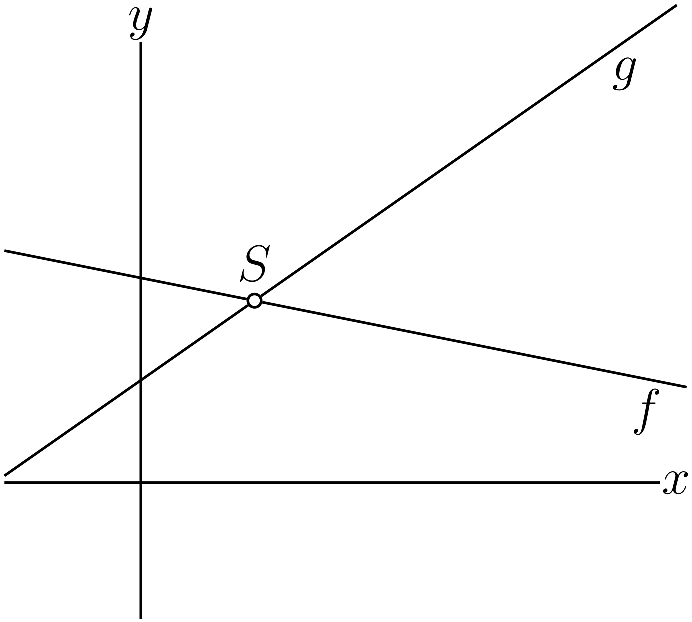
$g_1^2 + g_2^2 \neq 0$: eigentliche Gerade

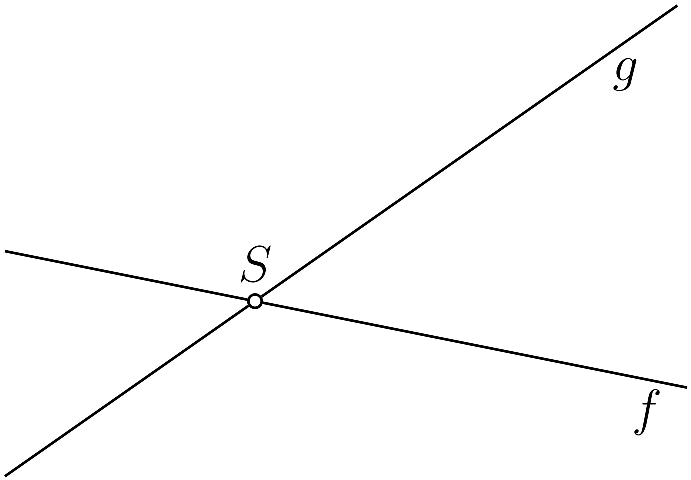


$$g\colon 7x - 10y + 15 = 0$$

$$f\colon x + 5y + 15 = 0$$

$$S\left(\tfrac{5}{3}, \tfrac{8}{3}\right)$$





$$g\colon 7x - 10y + 15 = 0$$

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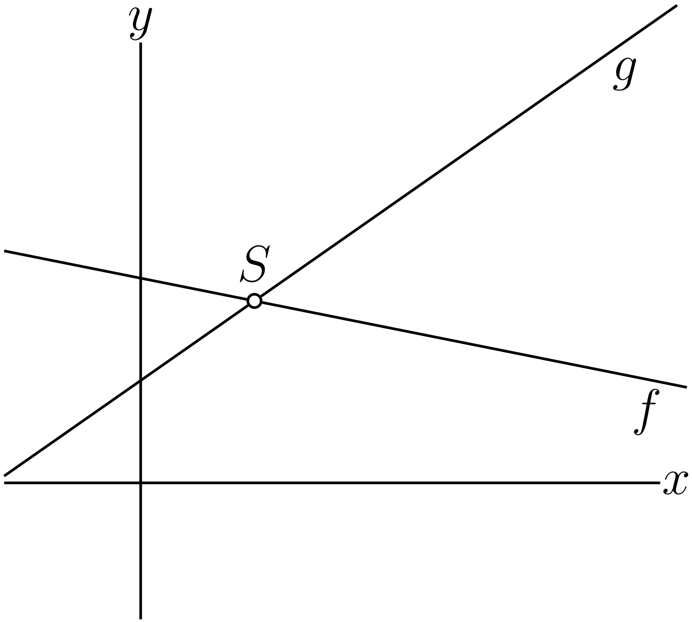
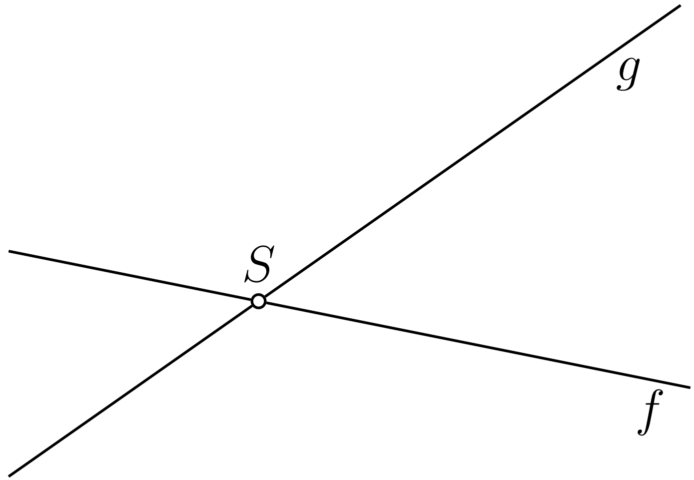
$$S\left(\tfrac{5}{3}, \tfrac{8}{3}\right)$$

$$g\colon 7x - 10y + 15z = 0$$

$$f\colon x + 5y + 15z = 0$$

$$S(5, 8, 3)$$





$$g\colon 7x - 10y + 15 = 0$$

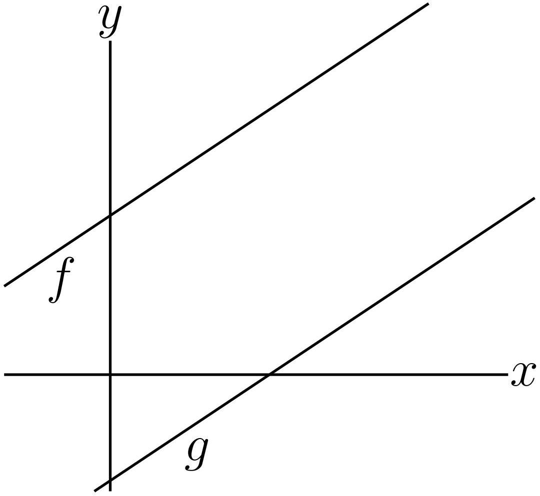
$$f\colon x + 5y + 15 = 0$$

$$S\left(\tfrac{5}{3}, \tfrac{8}{3}\right)$$

$$g\colon 7x - 10y + 15z = 0$$

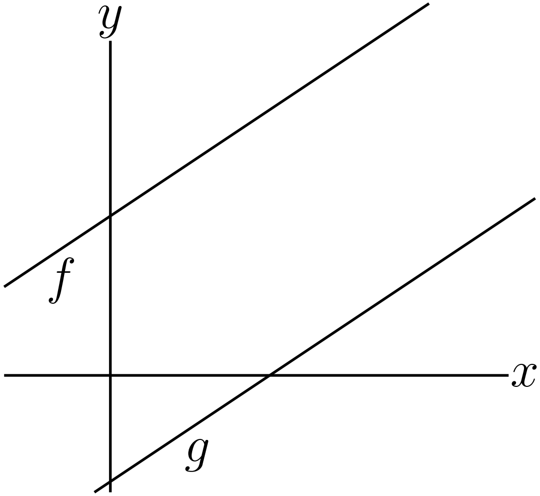
$$f\colon x + 5y + 15z = 0$$

$$S(5, 8, 3) \equiv \left(\tfrac{5}{3}, \tfrac{8}{3}, 1\right)$$



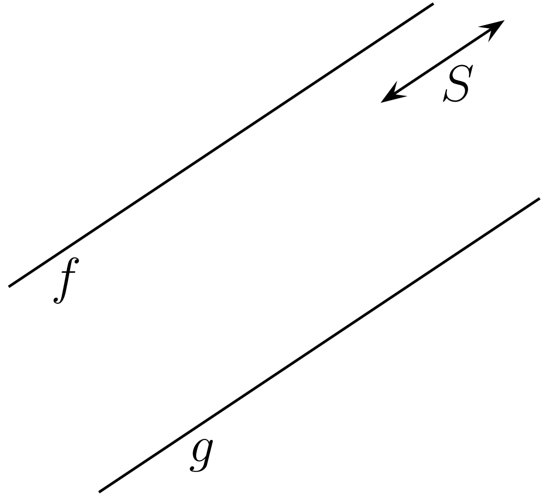
$$g\colon 2x - 3y - 6 = 0$$

$$f\colon 2x - 3y + 9 = 0$$



$$g\colon 2x - 3y - 6 = 0$$

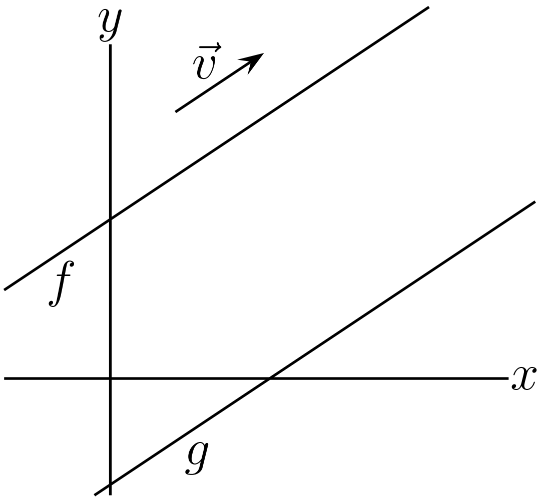
$$f\colon 2x - 3y + 9 = 0$$



$$g\colon 2x - 3y - 6z = 0$$

$$f\colon 2x - 3y + 9z = 0$$

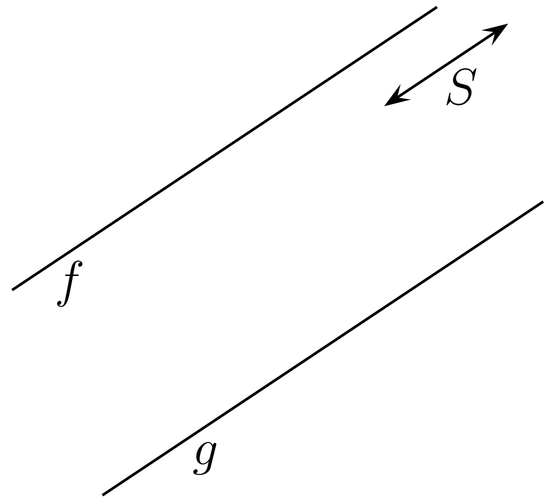
$$S(3, 2, 0)$$



$$g\colon 2x - 3y - 6 = 0$$

$$f\colon 2x - 3y + 9 = 0$$

$$\vec{v} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$



$$g\colon 2x - 3y - 6z = 0$$

$$f\colon 2x - 3y + 9z = 0$$

$$S(3, 2, 0)$$

Analytische Geometrie

Punkt (p_1, p_2, p_3)	Gerade $g_1 x + g_2 y + g_3 z = 0$

Analytische Geometrie

Punkt (p_1, p_2, p_3)

Datenstruktur: $\{p_1, p_2, p_3\}$

Gerade $g_1 x + g_2 y + g_3 z = 0$

Datenstruktur: $\{g_1, g_2, g_3\}$

Analytische Geometrie

Punkt (p_1, p_2, p_3)

Datenstruktur: $\{p_1, p_2, p_3\}$

Verbindungsgerade zweier Punkte
$A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$

Gerade $g_1 x + g_2 y + g_3 z = 0$

Datenstruktur: $\{g_1, g_2, g_3\}$

Schnittpunkt zweier Geraden
$g = \{g_1, g_2, g_3\}$, $f = \{f_1, f_2, f_3\}$

Analytische Geometrie

Punkt (p_1, p_2, p_3)

Datenstruktur: $\{p_1, p_2, p_3\}$

Verbindungsgerade zweier Punkte
$A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$

$\{a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1\}$

Gerade $g_1 x + g_2 y + g_3 z = 0$

Datenstruktur: $\{g_1, g_2, g_3\}$

Schnittpunkt zweier Geraden
$g = \{g_1, g_2, g_3\}$, $f = \{f_1, f_2, f_3\}$

$\{g_2 f_3 - g_3 f_2,\ g_3 f_1 - g_1 f_3,\ g_1 f_2 - g_2 f_1\}$

Analytische Geometrie

Punkt (p_1, p_2, p_3)

Datenstruktur: $\{p_1, p_2, p_3\}$

Verbindungsgerade zweier Punkte
$A = \{a_1, a_2, a_3\}$, $B = \{b_1, b_2, b_3\}$

$\{a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1\}$

$= \{a_1, a_2, a_3\} \times \{b_1, b_2, b_3\}$

Gerade $g_1 x + g_2 y + g_3 z = 0$

Datenstruktur: $\{g_1, g_2, g_3\}$

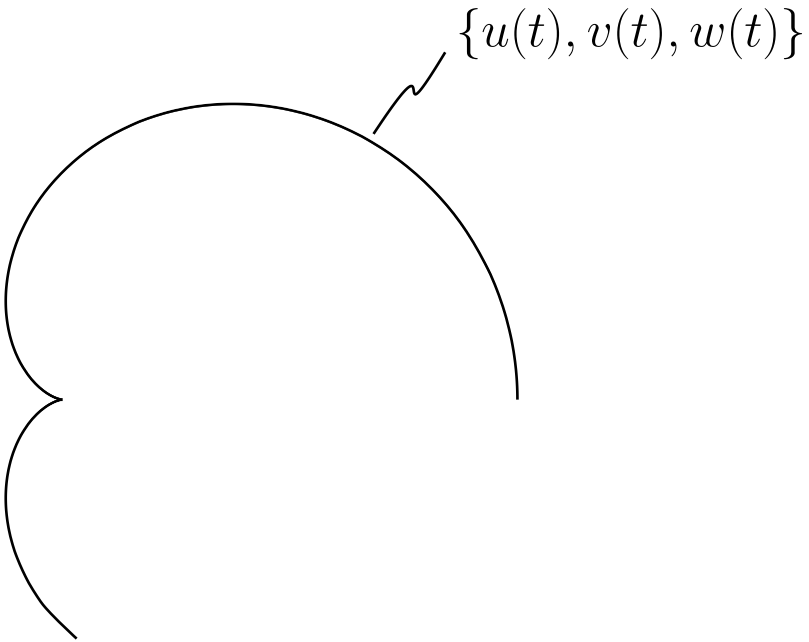
Schnittpunkt zweier Geraden
$g = \{g_1, g_2, g_3\}$, $f = \{f_1, f_2, f_3\}$

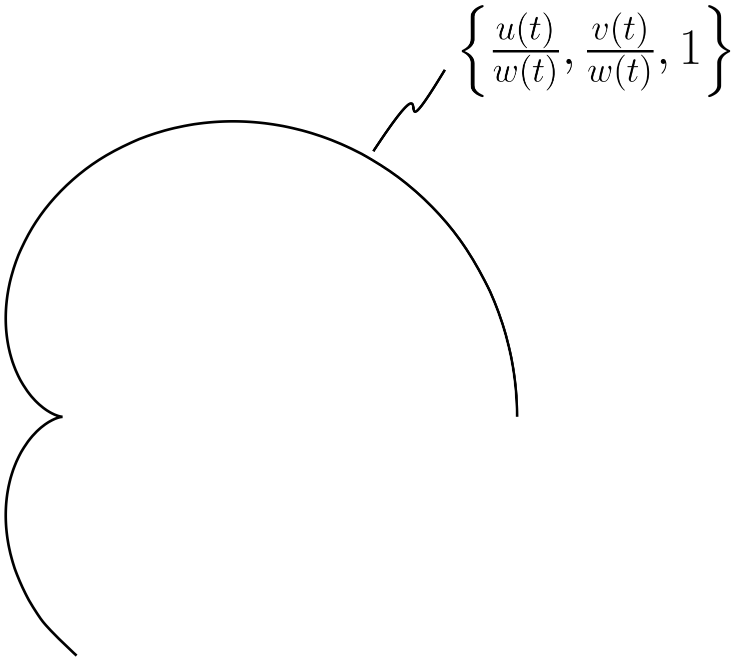
$\{g_2 f_3 - g_3 f_2,\ g_3 f_1 - g_1 f_3,\ g_1 f_2 - g_2 f_1\}$

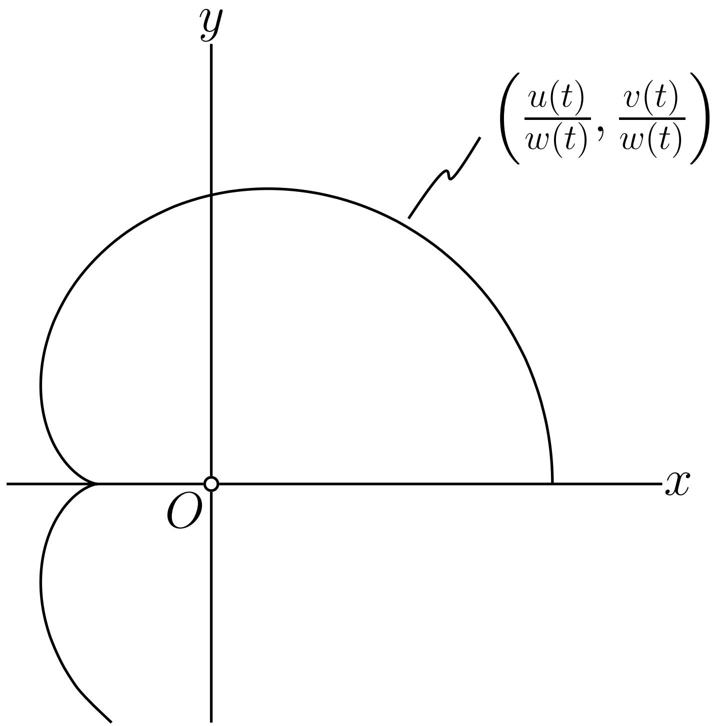
$= \{g_1, g_2, g_3\} \times \{f_1, f_2, f_3\}$

Formulierung in Mathematica:

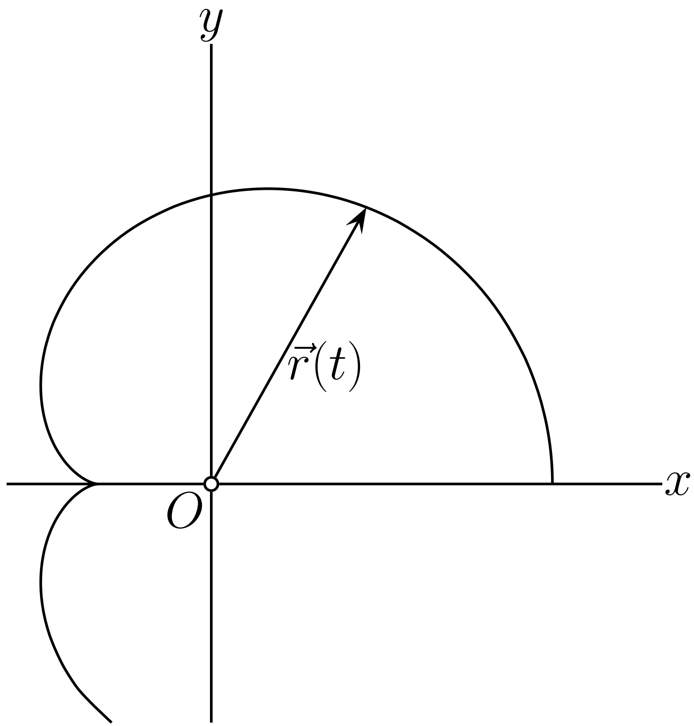
```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];
```


$\{u(t), v(t), w(t)\}$

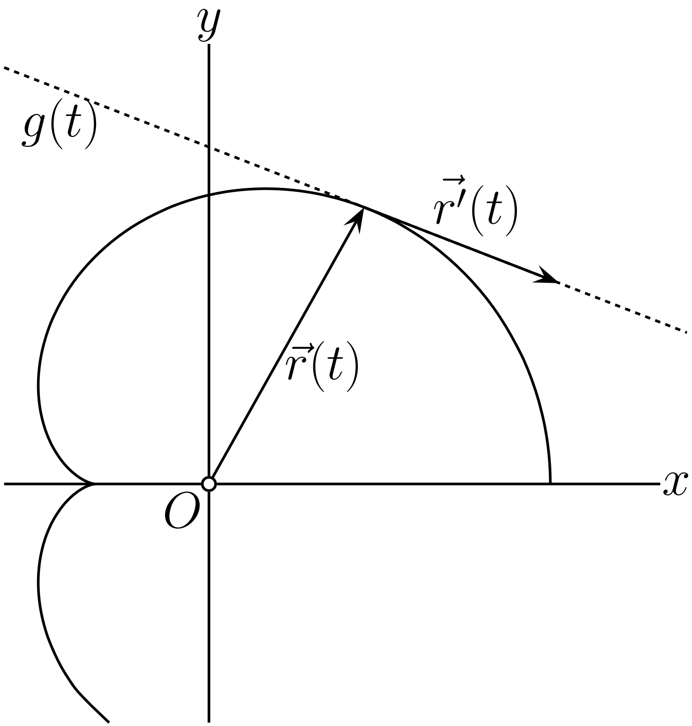

$\left\{\frac{u(t)}{w(t)}, \frac{v(t)}{w(t)}, 1\right\}$



y
$\left(\frac{u(t)}{w(t)}, \frac{v(t)}{w(t)}\right)$
x
O

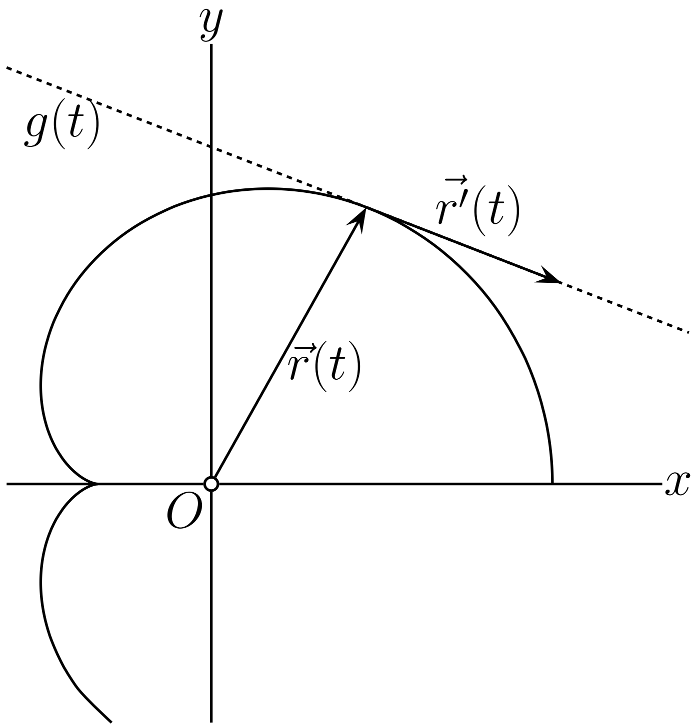


$$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u(t)/w(t) \\ v(t)/w(t) \end{bmatrix}$$



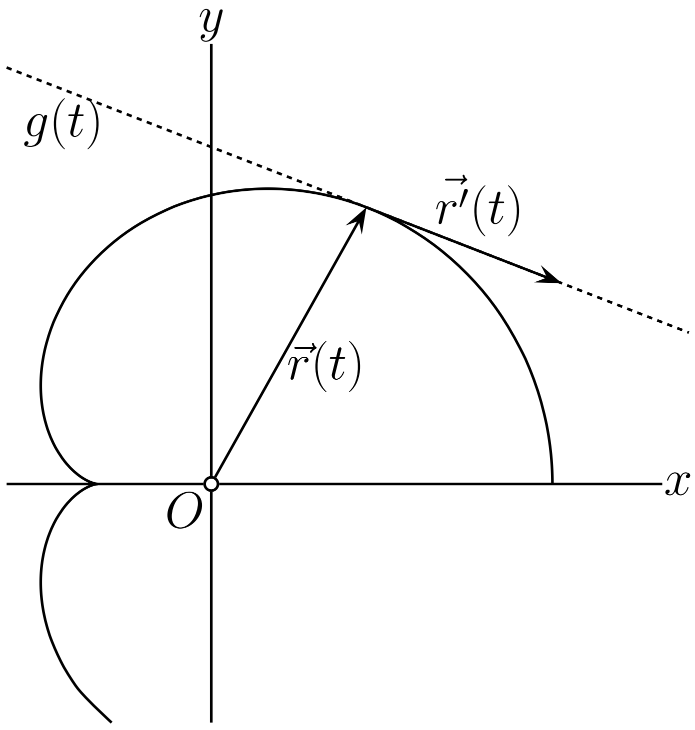
$$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u(t)/w(t) \\ v(t)/w(t) \end{bmatrix}$$

$$\vec{r}'(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix}$$



$$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u(t)/w(t) \\ v(t)/w(t) \end{bmatrix}$$

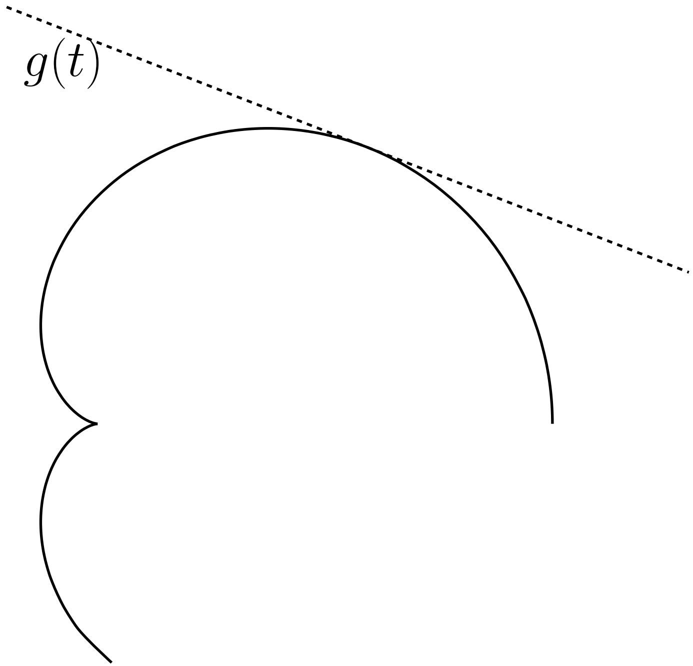
$$\vec{r}'(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} (u'w - uw')/w^2 \\ (v'w - vw')/w^2 \end{bmatrix}$$



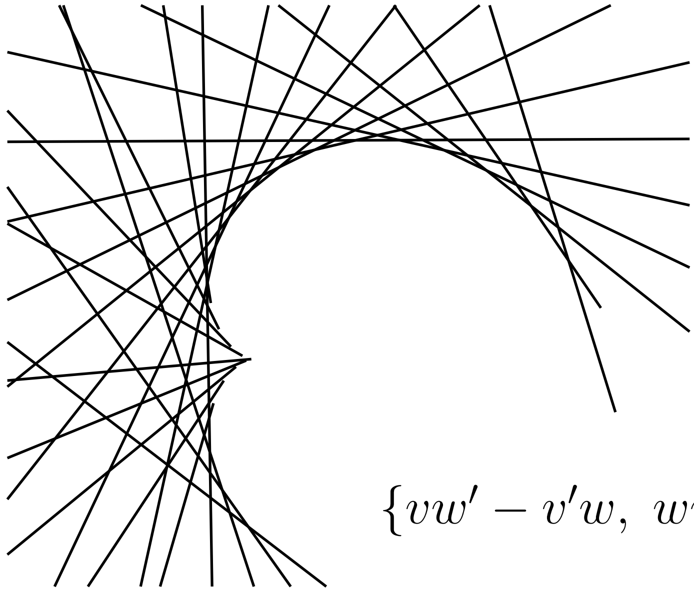
$$\vec{r}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} u(t)/w(t) \\ v(t)/w(t) \end{bmatrix}$$

$$\vec{r}'(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} (u'w - uw')/w^2 \\ (v'w - vw')/w^2 \end{bmatrix}$$

$$g(t)\colon (v'w - vw')x + (w'u - wu')y + (u'v - uv') \quad = \quad 0$$



$$g(t)\colon (v'w - vw')x + (w'u - wu')y + (u'v - uv')z \;=\; 0$$



$$\{vw' - v'w,\ wu' - w'u,\ uv' - u'v\}$$

Analytische Geometrie

Ortskurve $\{u(t), v(t), w(t)\}$

als Hüllkurve:
$\{vw' - v'w,\ wu' - w'u,\ uv' - u'v\}$

Analytische Geometrie

Ortskurve $\{u(t), v(t), w(t)\}$

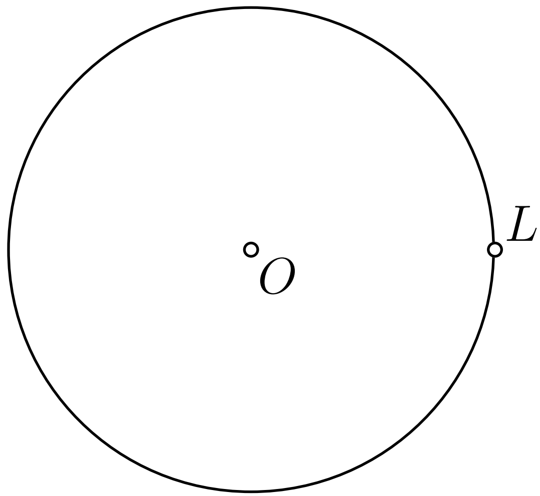
als Hüllkurve:
$\{u, v, w\} \times \{u', v', w'\}$

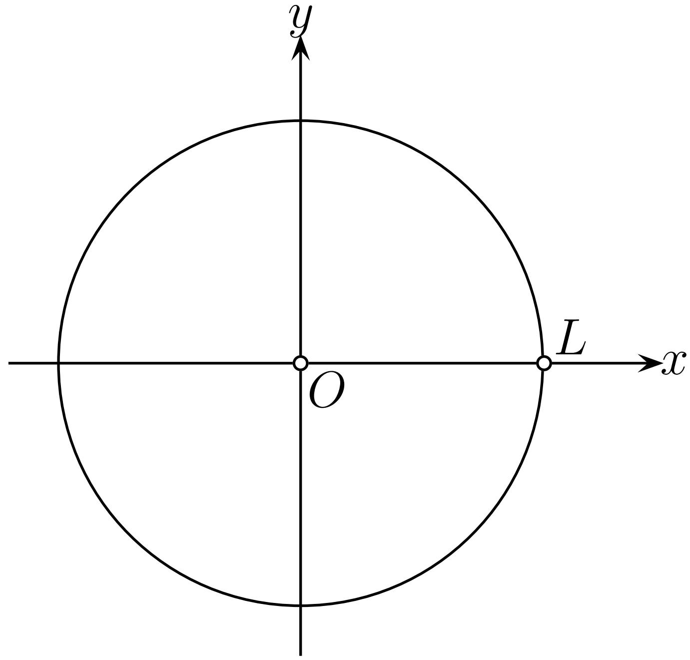
Analytische Geometrie

Ortskurve $\{u(t), v(t), w(t)\}$	Hüllkurve $\{a(t), b(t), c(t)\}$
als Hüllkurve: $\{u, v, w\} \times \{u', v', w'\}$	als Ortskurve: $\{a, b, c\} \times \{a', b', c'\}$

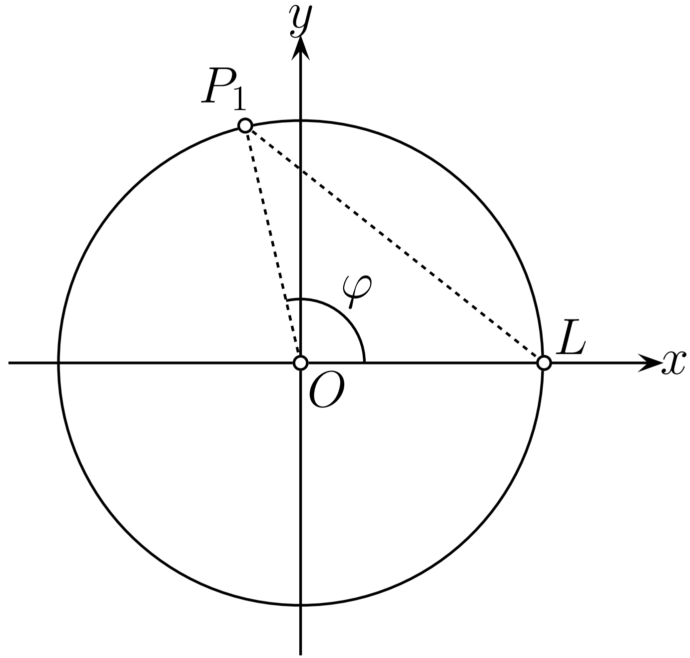
Formulierung in Mathematica:

```
Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];
```

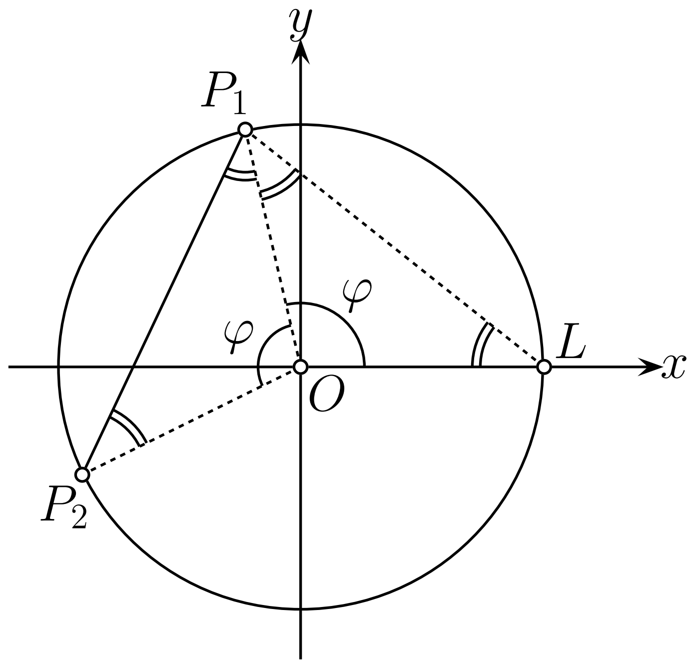

O
L



y
L
x
O

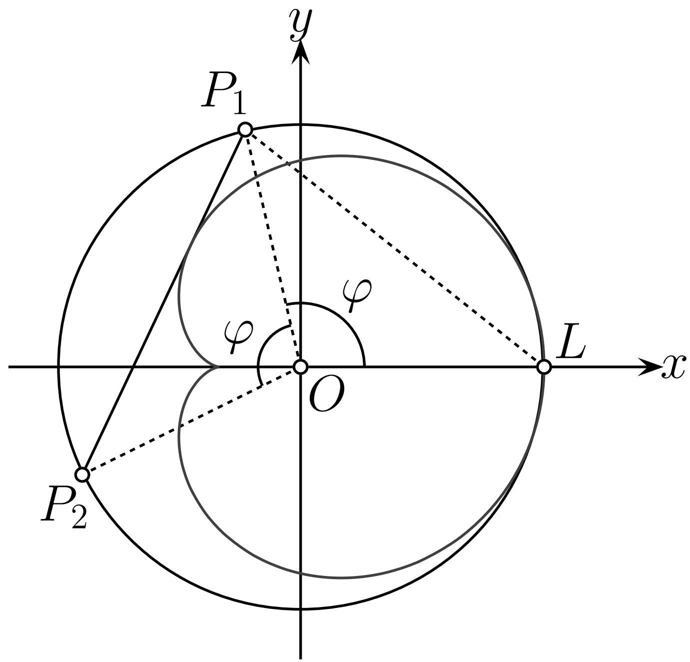


$P_1\,(\cos(\varphi), \sin(\varphi))$



$P_1\,(\cos(\varphi), \sin(\varphi))$

$P_2\,(\cos(2\varphi), \sin(2\varphi))$



$P_1\,(\cos(\varphi), \sin(\varphi))$

$P_2\,(\cos(2\varphi), \sin(2\varphi))$

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_] = {Cos[φ], Sin[φ], 1};
g[φ_] = Verb[p[φ], p[2 φ]];
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_] = {Cos[φ], Sin[φ], 1};
g[φ_] = Verb[p[φ], p[2 φ]];

kardioide[φ_] = Ortskurve[g[φ], φ]
```

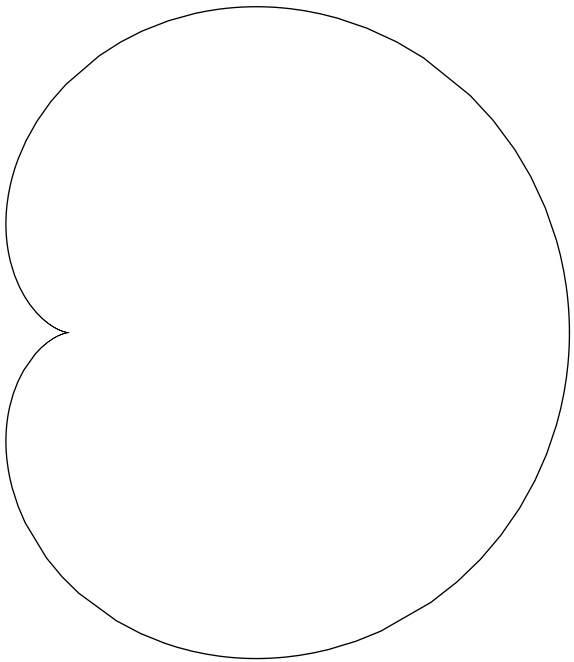
```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_] = {Cos[φ], Sin[φ], 1};
g[φ_] = Verb[p[φ], p[2 φ]];

kardioide[φ_] = Ortskurve[g[φ], φ] // Cart
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_] = {Cos[φ], Sin[φ], 1};
g[φ_] = Verb[p[φ], p[2 φ]];

kardioide[φ_] = Ortskurve[g[φ], φ] // Cart // FullSimplify
                                              // TrigReduce
```

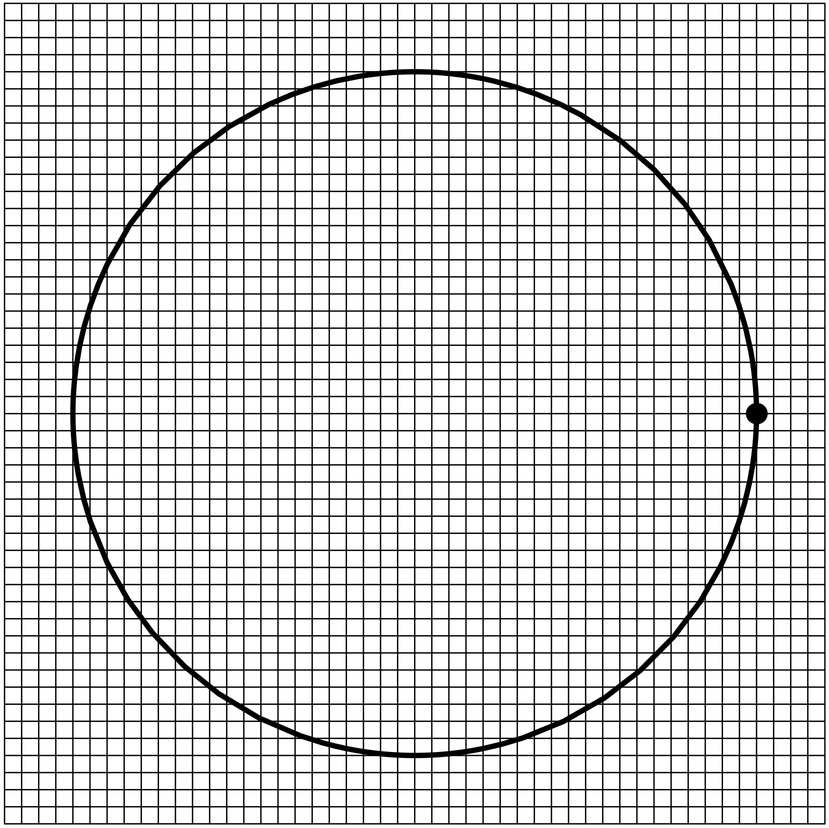
```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_] = {Cos[φ], Sin[φ], 1};
g[φ_] = Verb[p[φ], p[2 φ]];

kardioide[φ_] = Ortskurve[g[φ], φ] // Cart // FullSimplify
                                        // TrigReduce
```

$$\left\{\frac{1}{3}\,(2\,\mathrm{Cos}[\varphi]+\mathrm{Cos}[2\,\varphi]),\ \frac{1}{3}\,(2\,\mathrm{Sin}[\varphi]+\mathrm{Sin}[2\,\varphi])\right\}$$

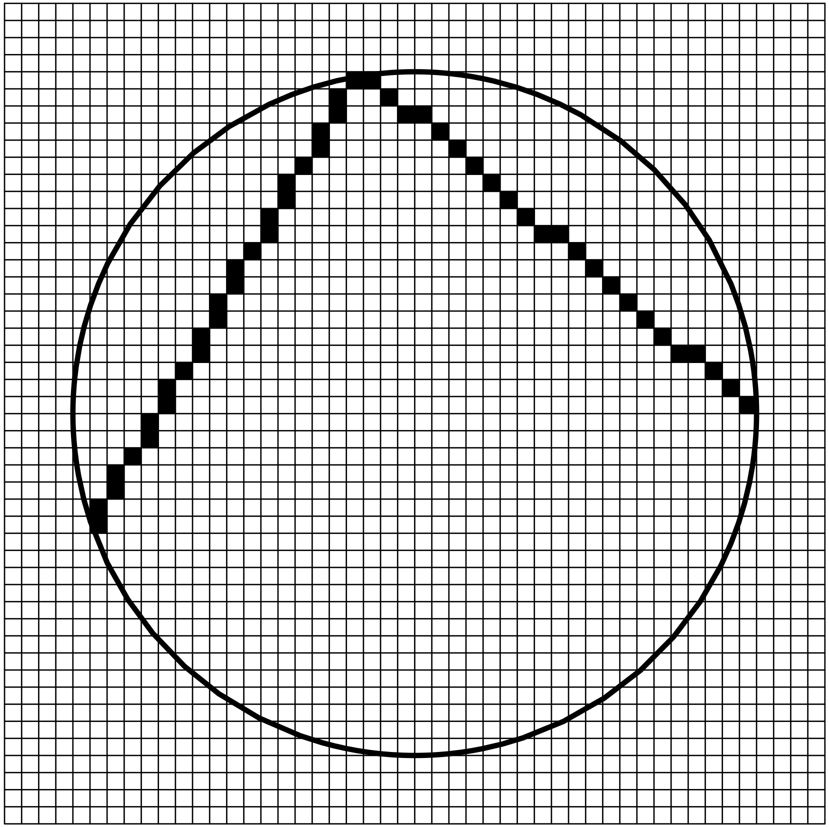
```
ParametricPlot[kardioide[φ], {φ, 0, 2 Pi},
                    AspectRatio → Automatic, Axes → False];
```



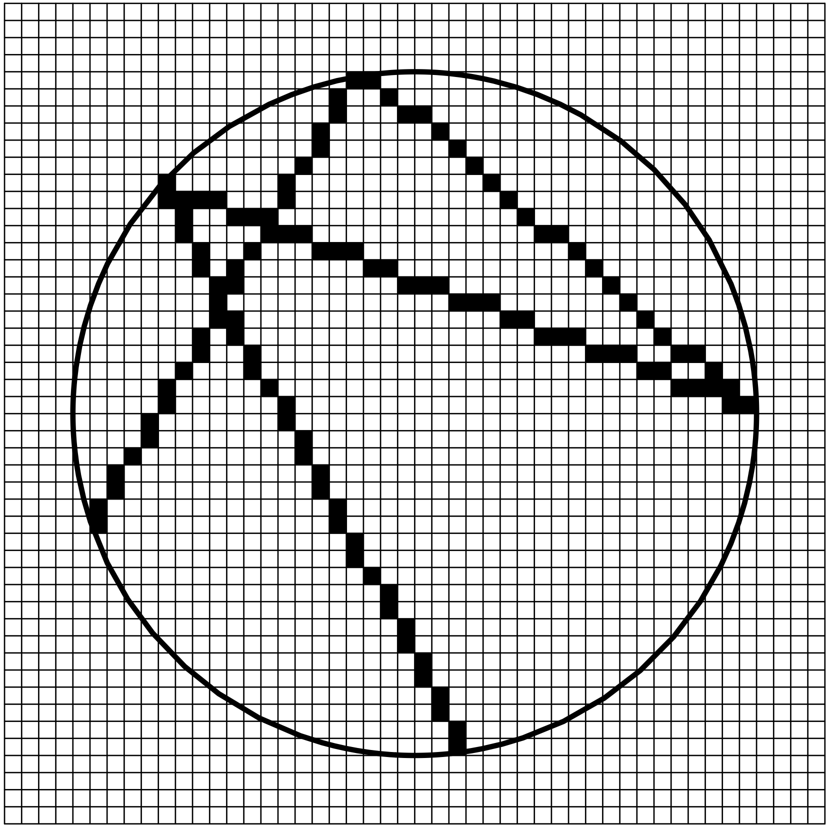
Lieberherr-Methode

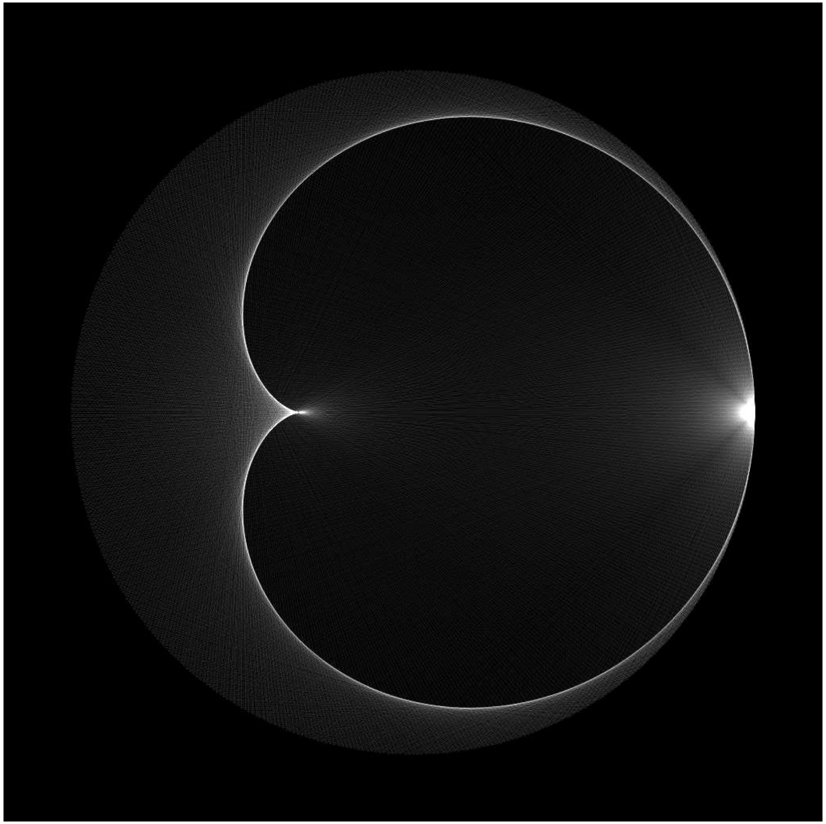


Lieberherr-Methode

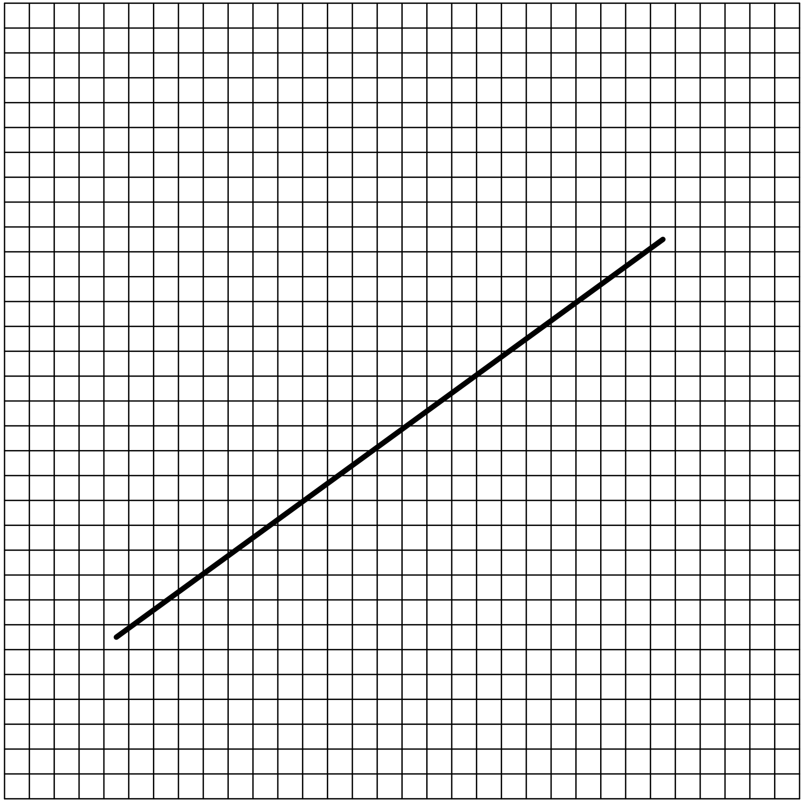


Lieberherr-Methode

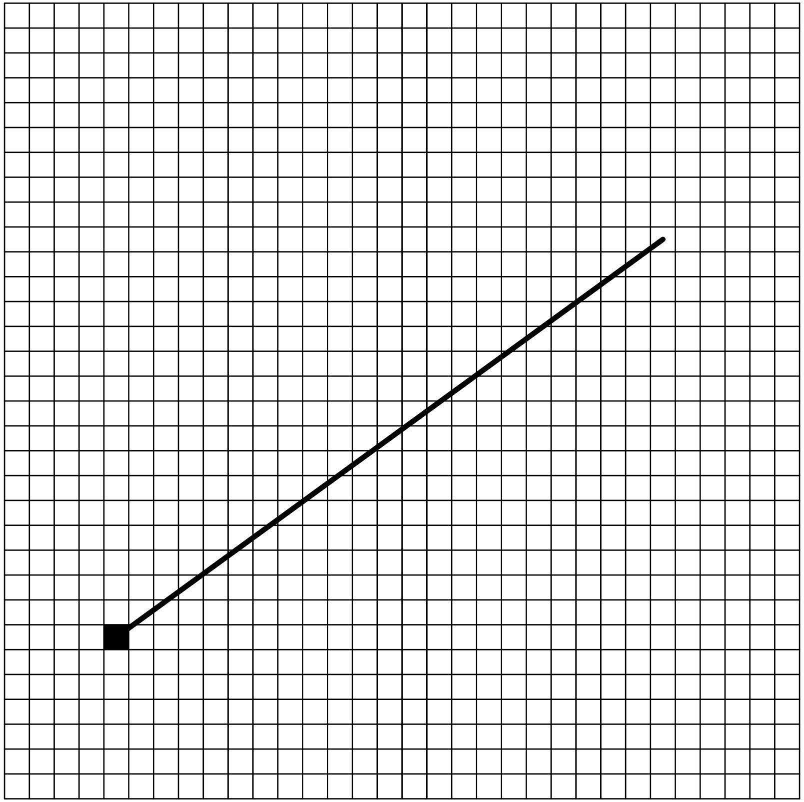




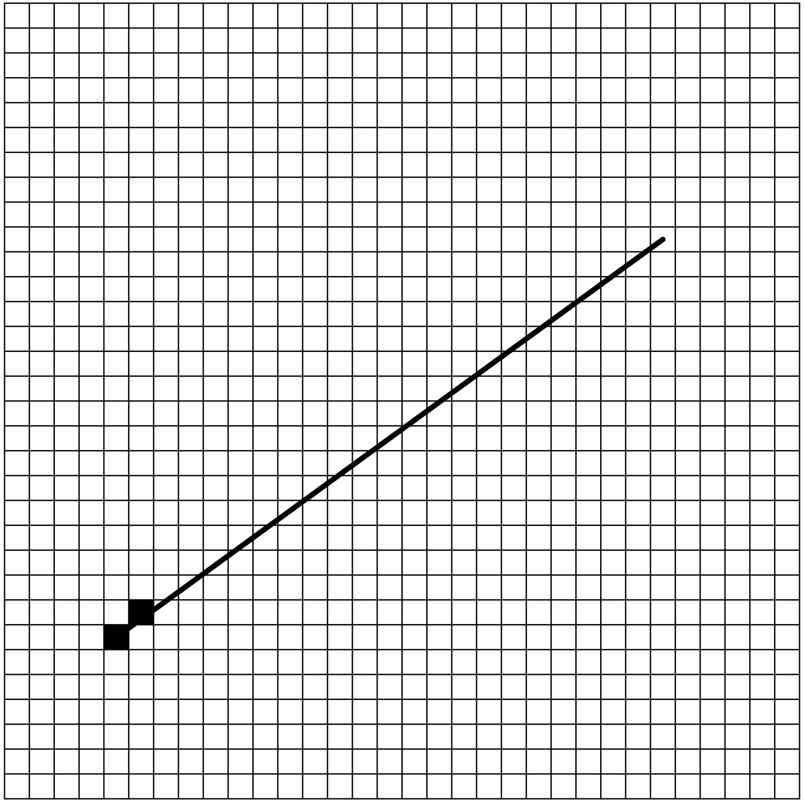
Bresenham-Algorithmus



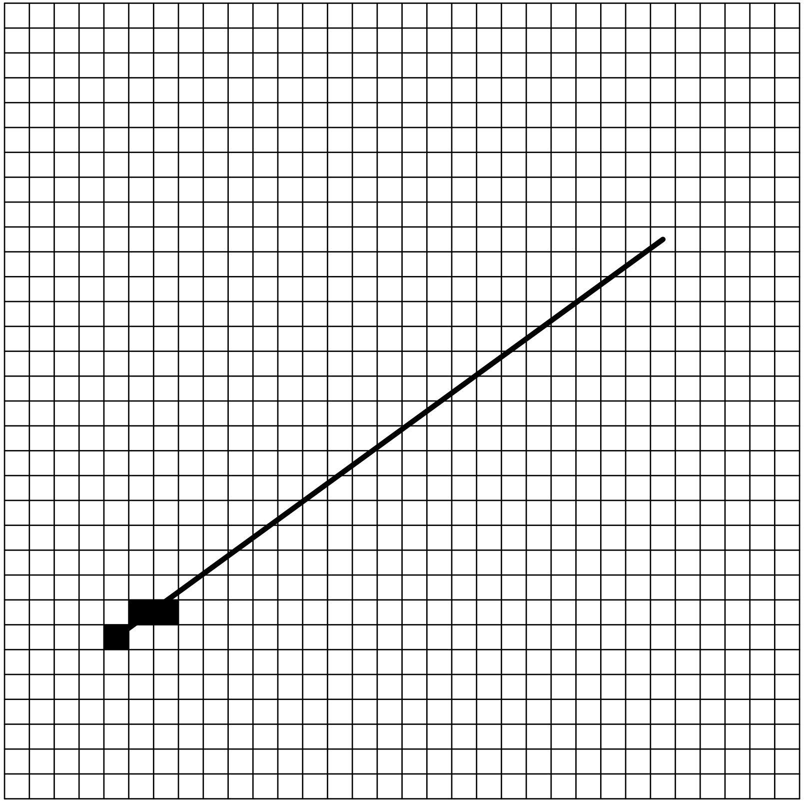
Bresenham-Algorithmus



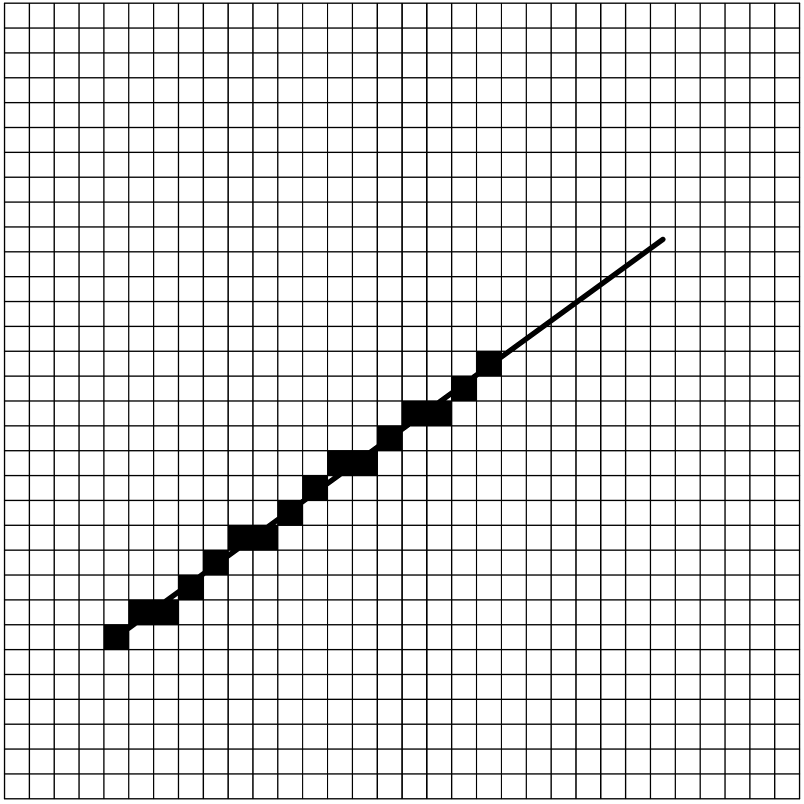
Bresenham-Algorithmus



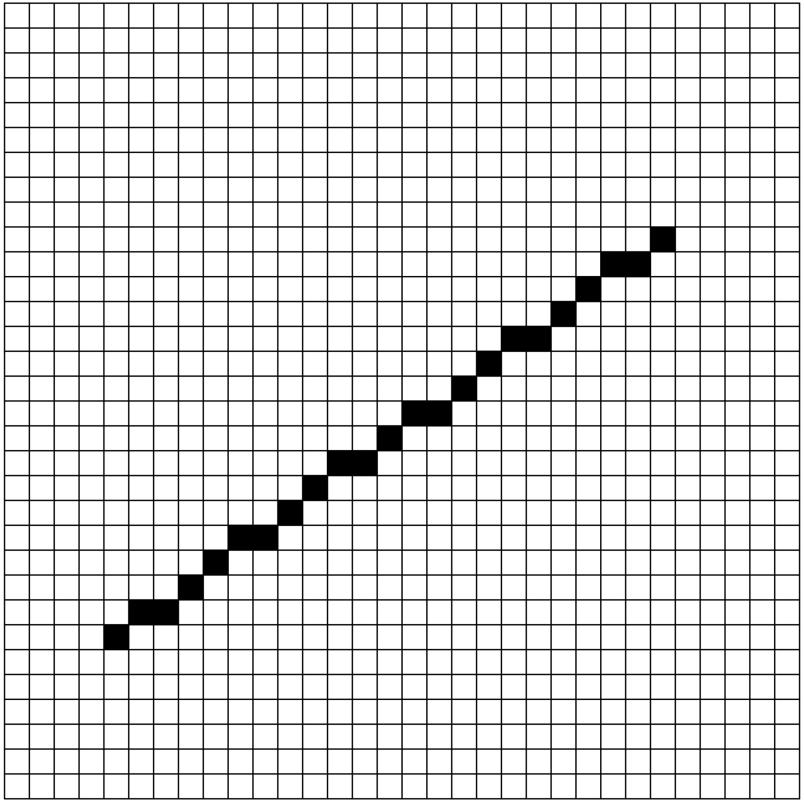
Bresenham-Algorithmus

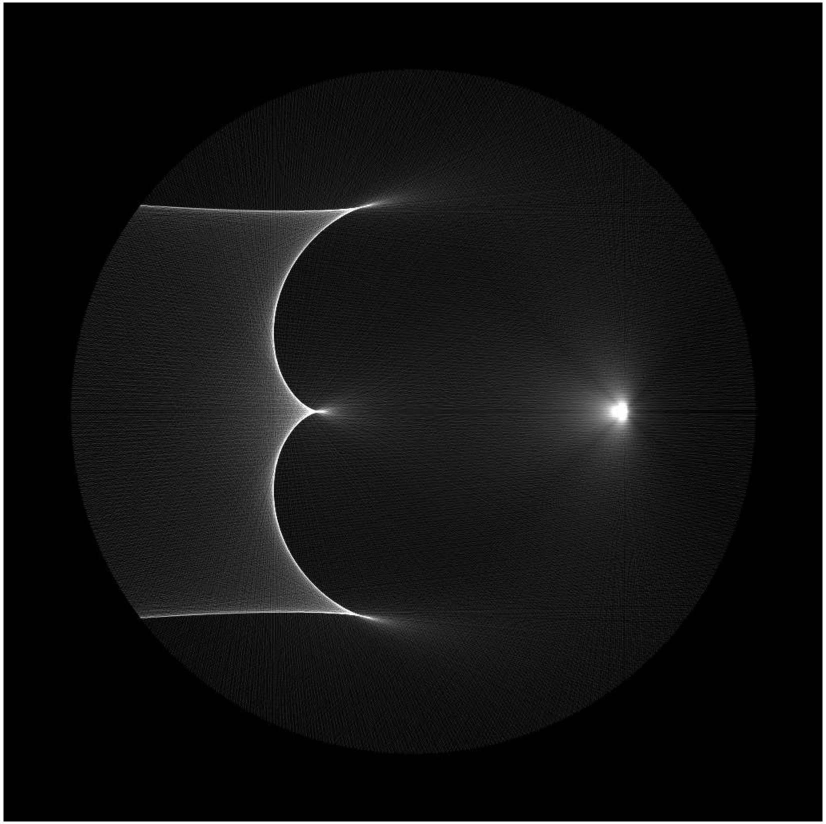


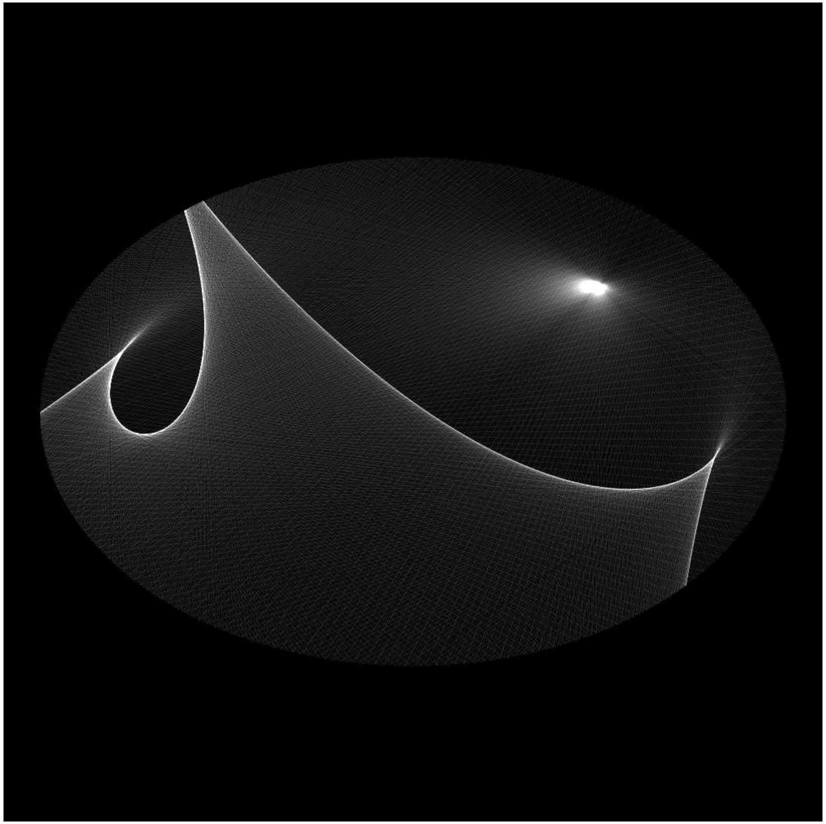
Bresenham-Algorithmus

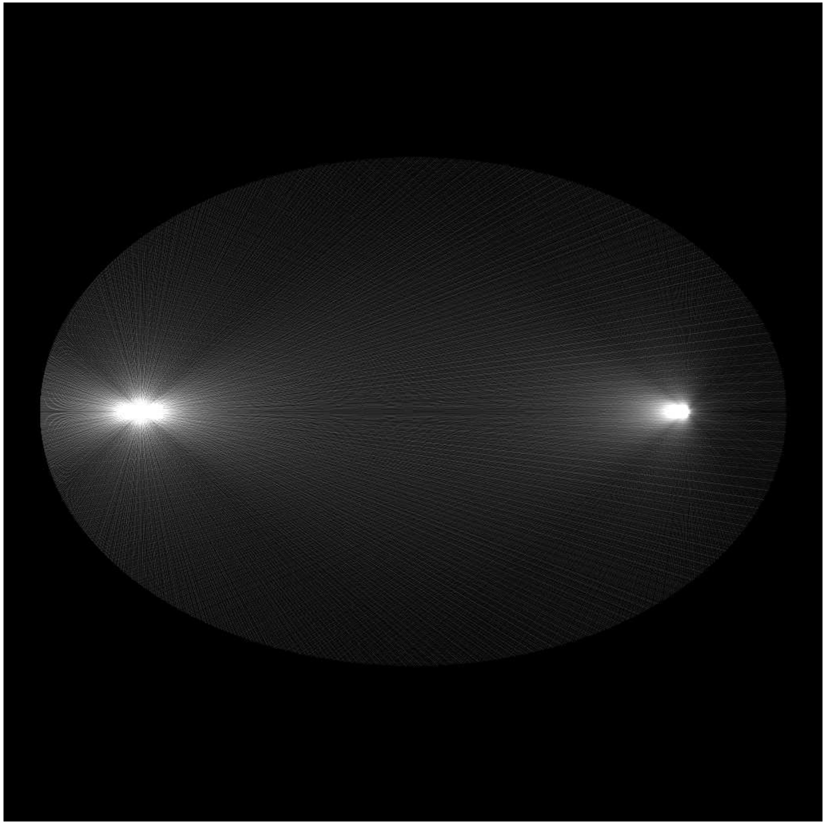


Bresenham-Algorithmus

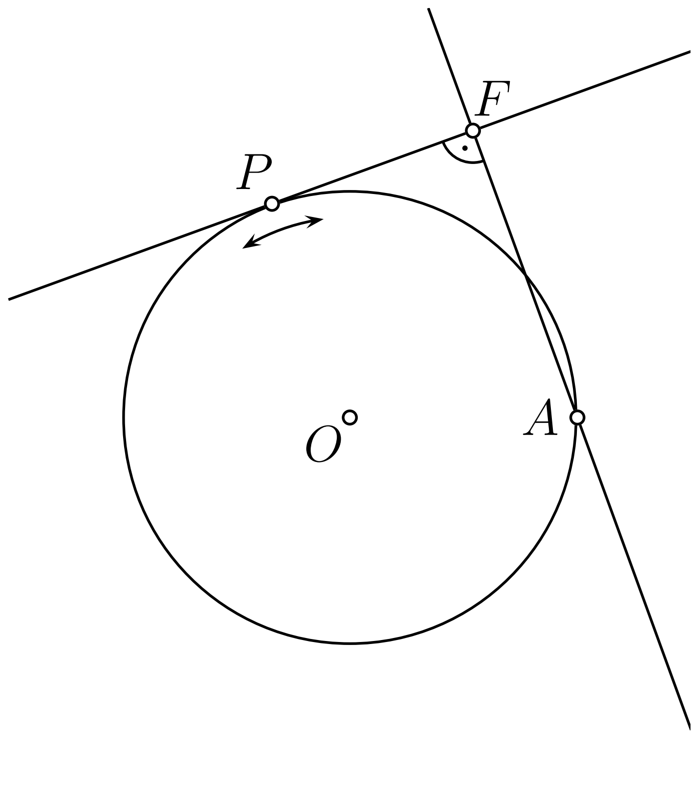




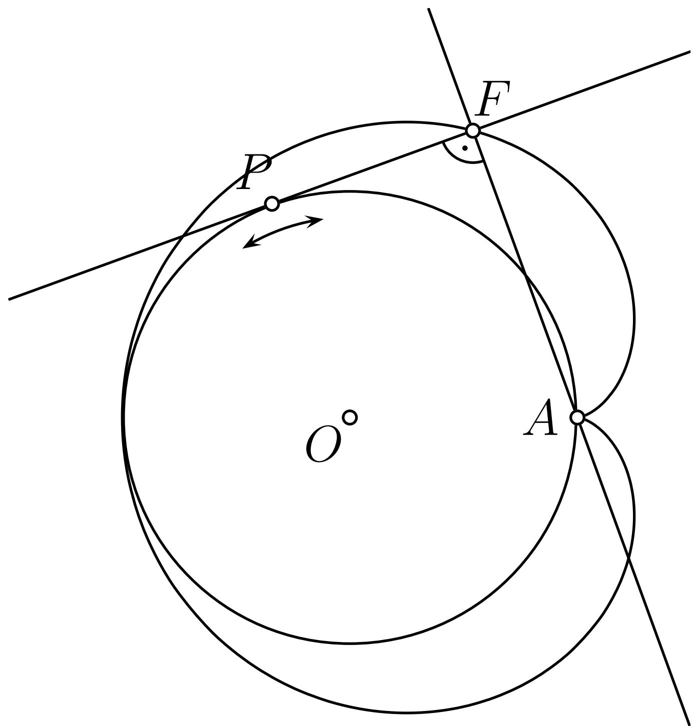




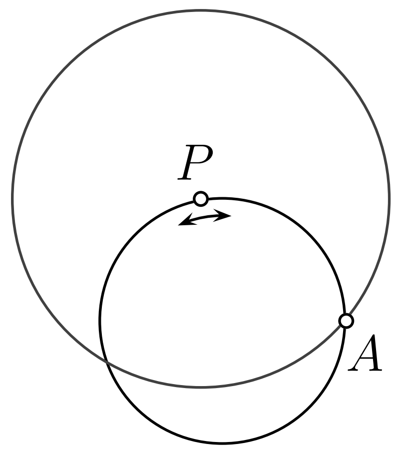
Die Kardioide als Fusspunktkurve



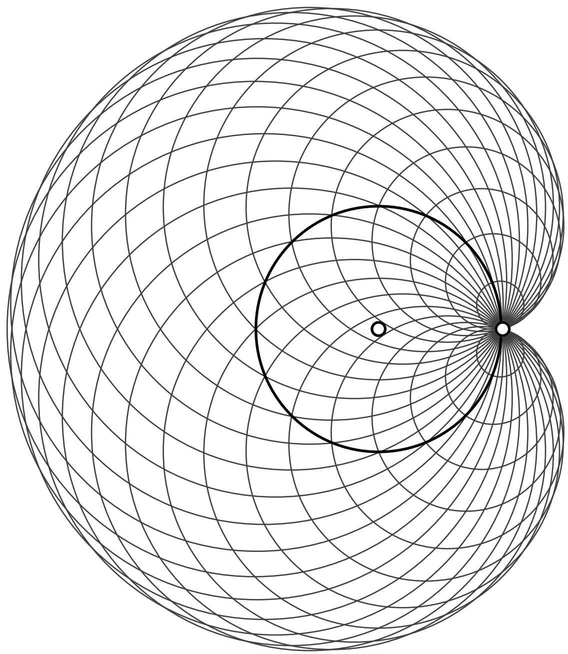
Die Kardioide als Fusspunktkurve



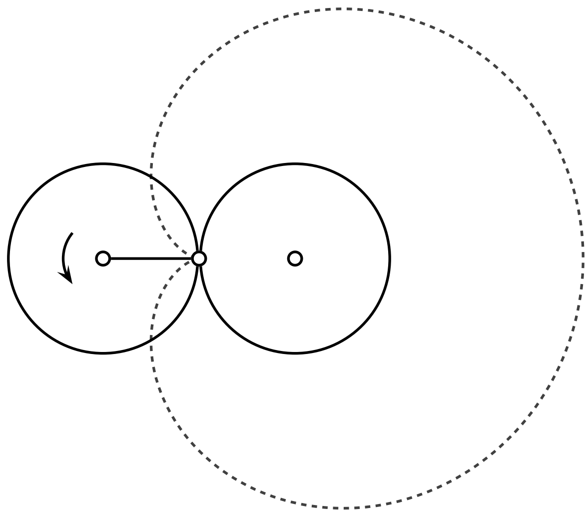
Die Kardioide als Hüllkurve von Kreisen



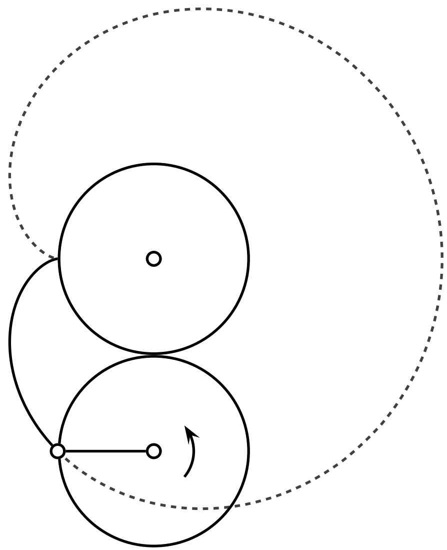
Die Kardioide als Hüllkurve von Kreisen



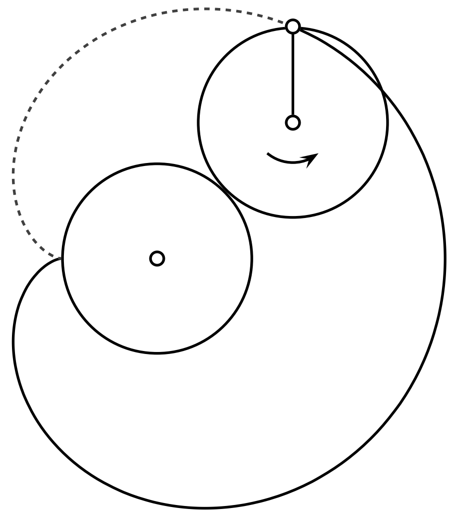
Die Kardioide als Rollkurve



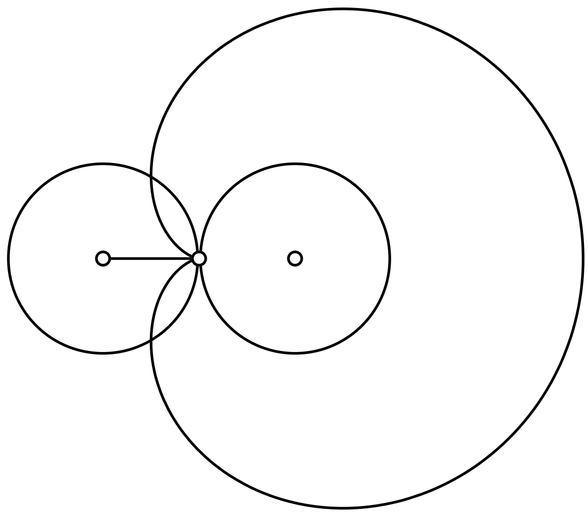
Die Kardioide als Rollkurve



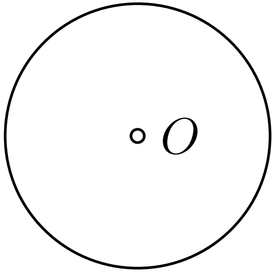
Die Kardioide als Rollkurve



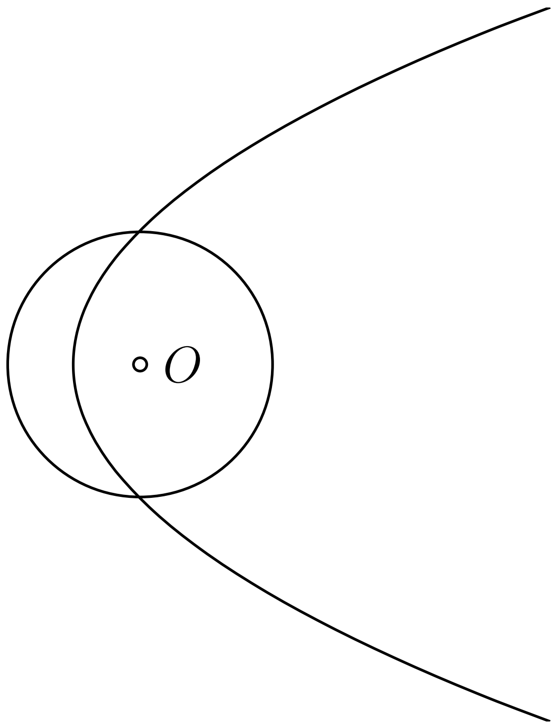
Die Kardioide als Rollkurve



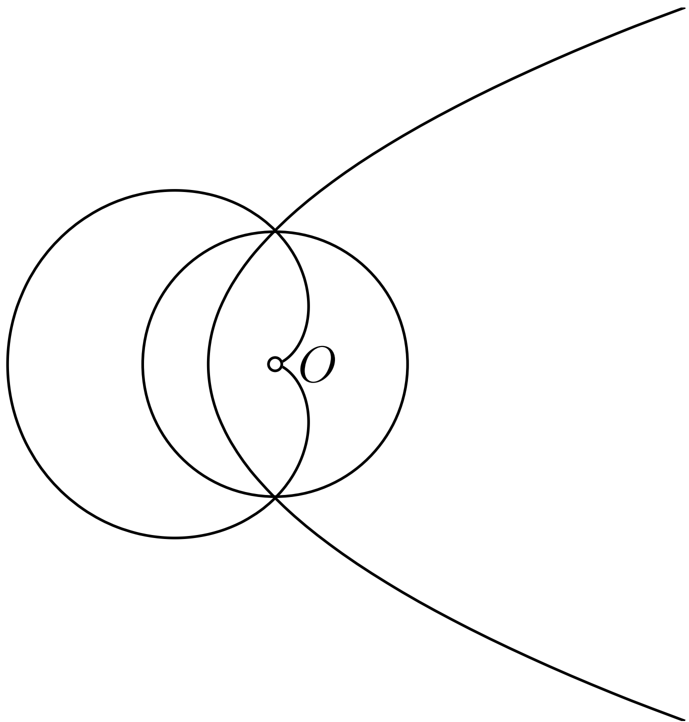
Die Kardioide als Inverse



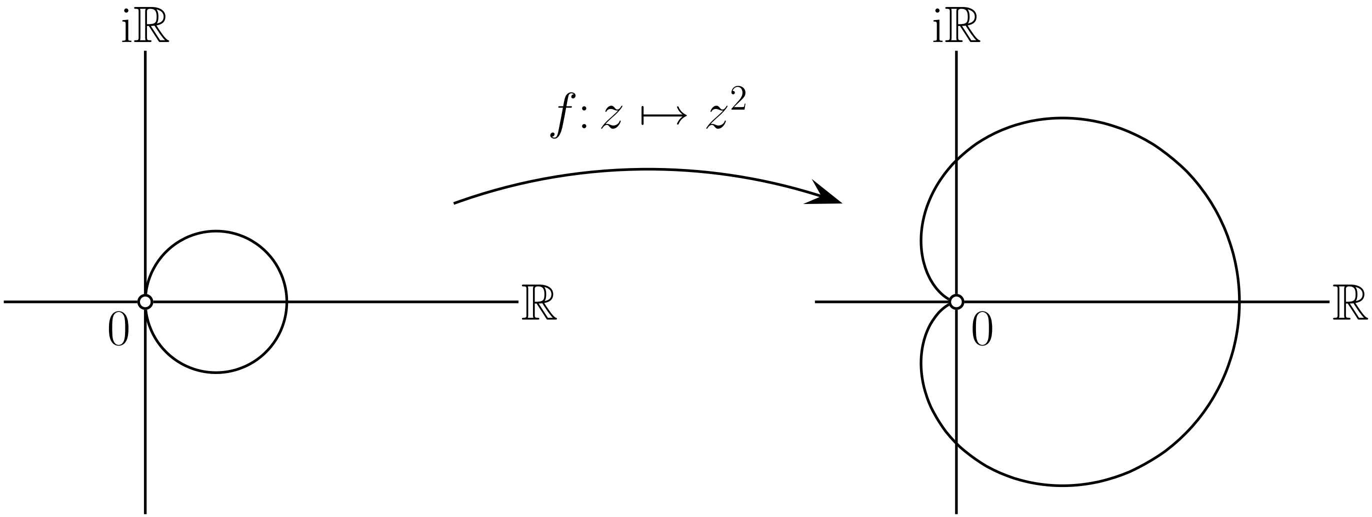
Die Kardioide als Inverse

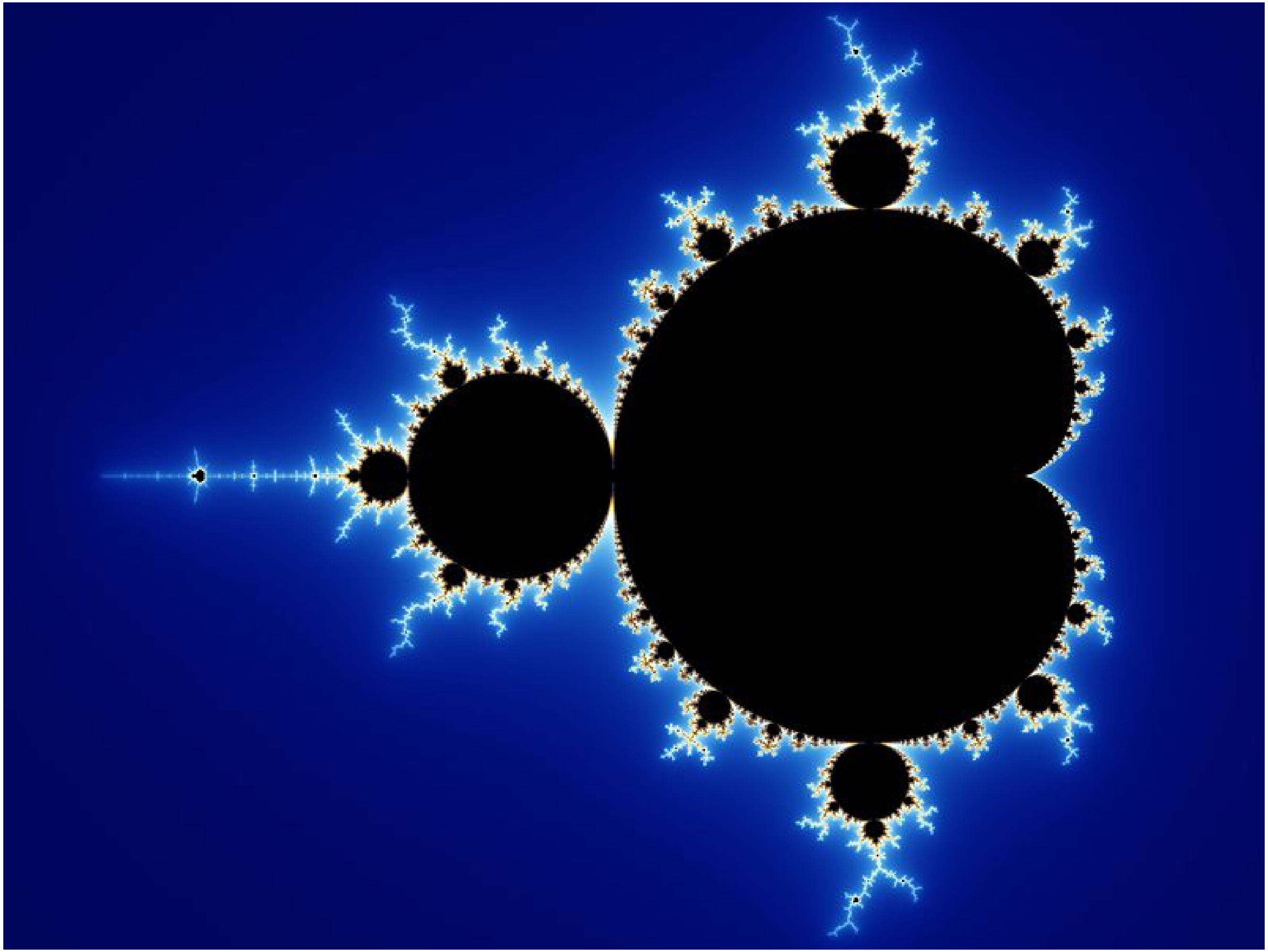


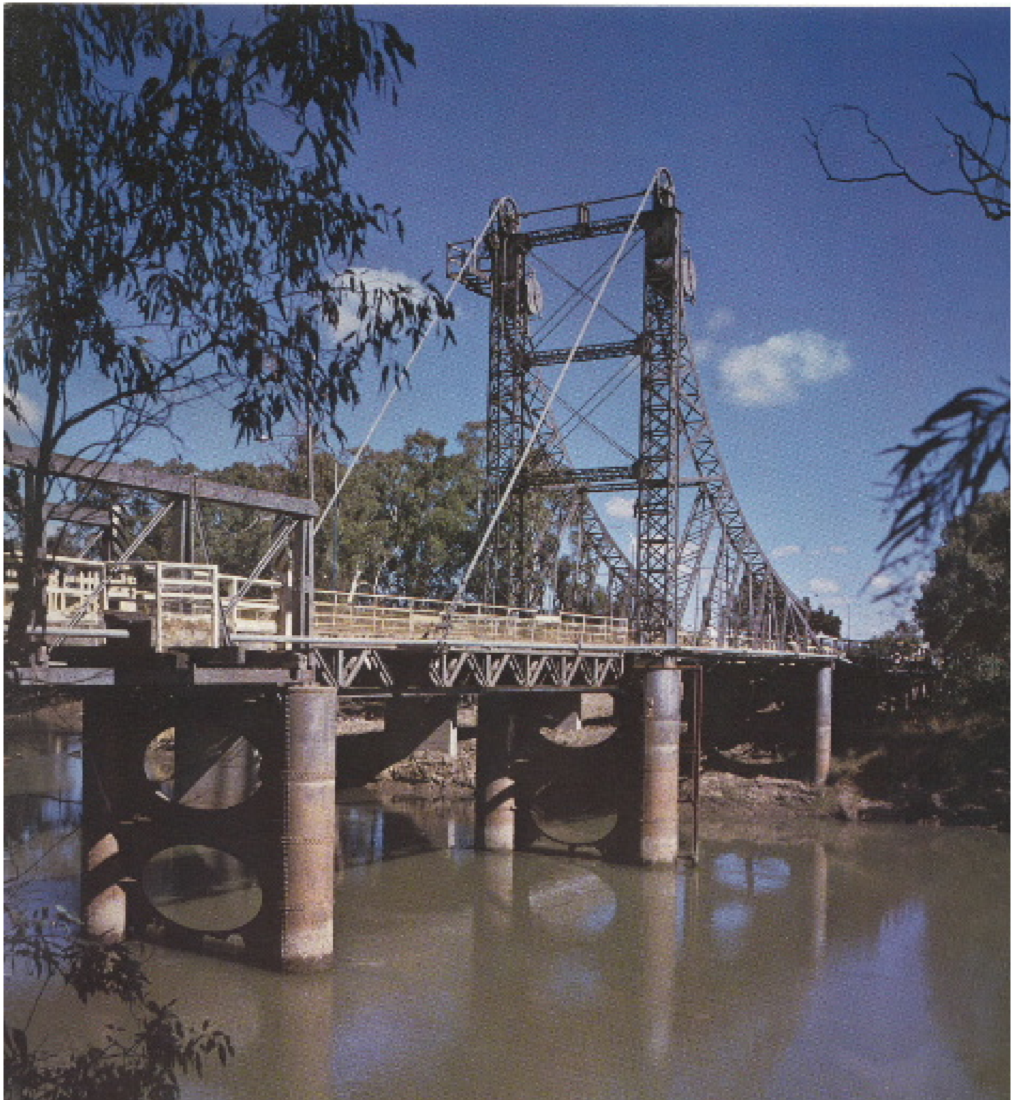
Die Kardioide als Inverse

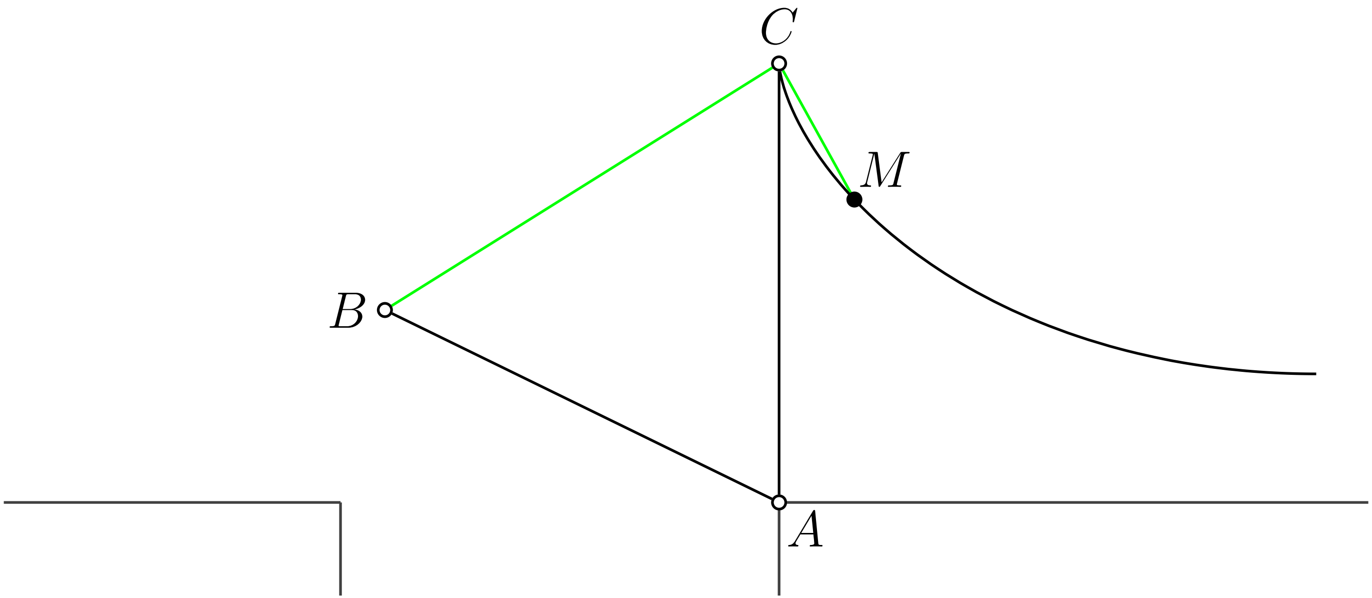


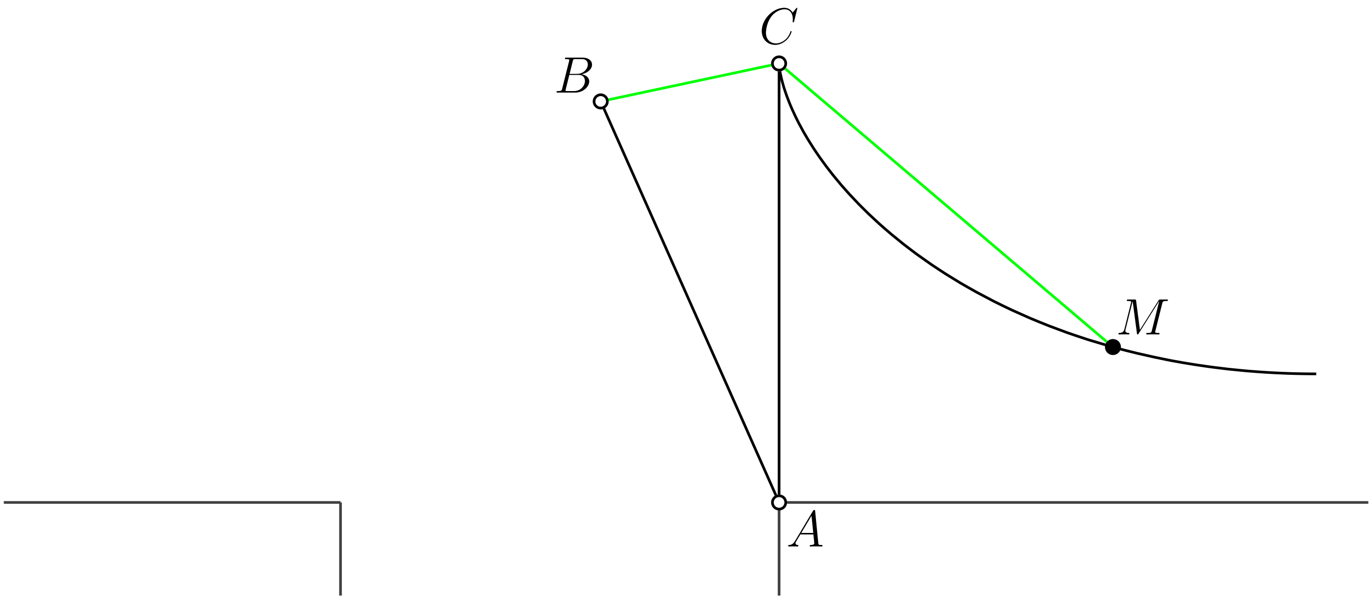
Die Kardioide als Bild $z \mapsto z^2$

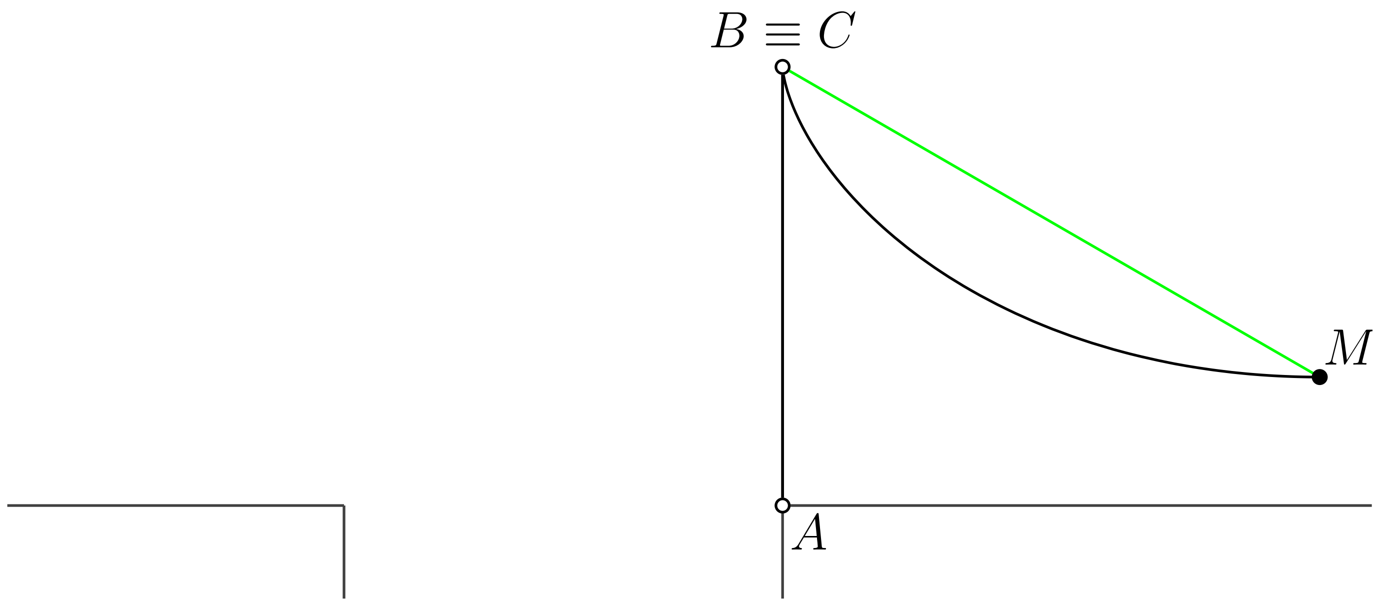


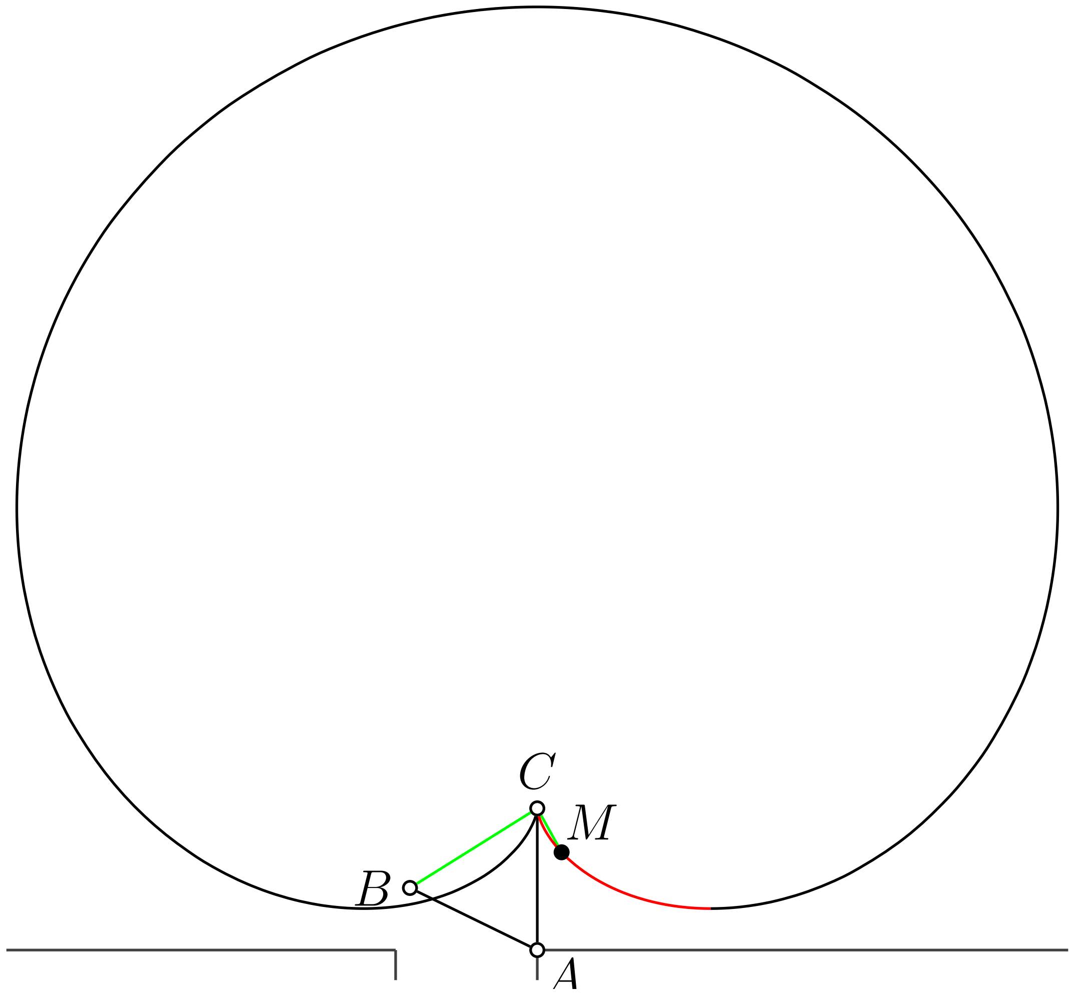




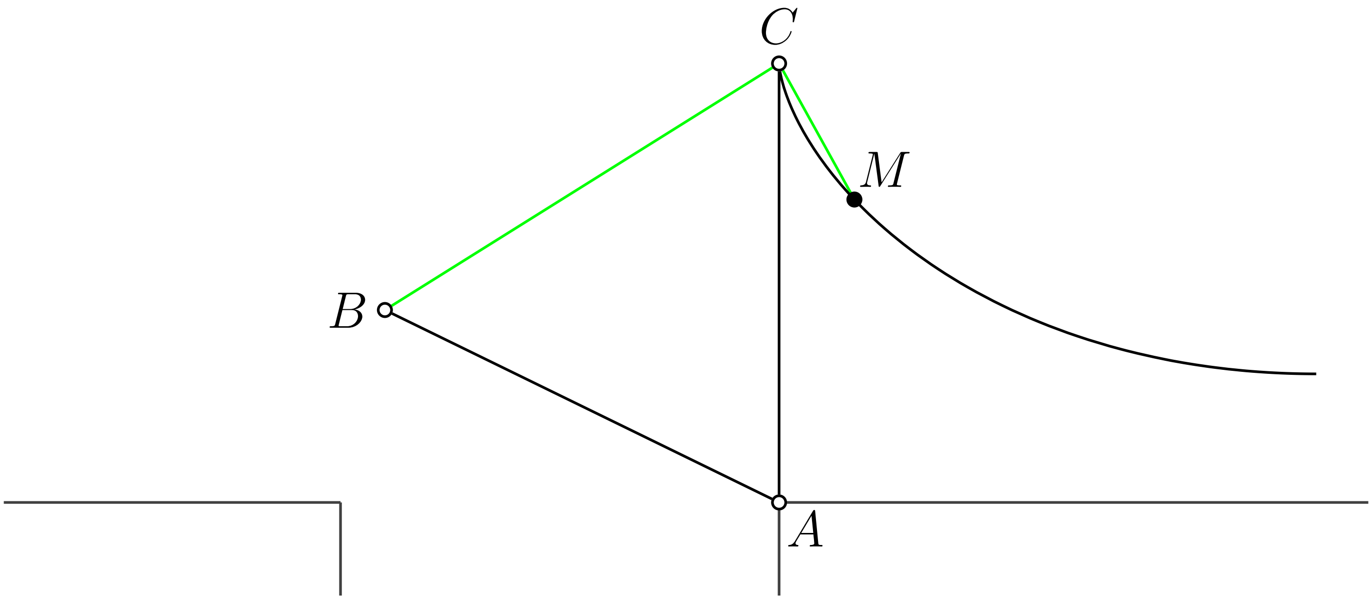

C
M
B
A

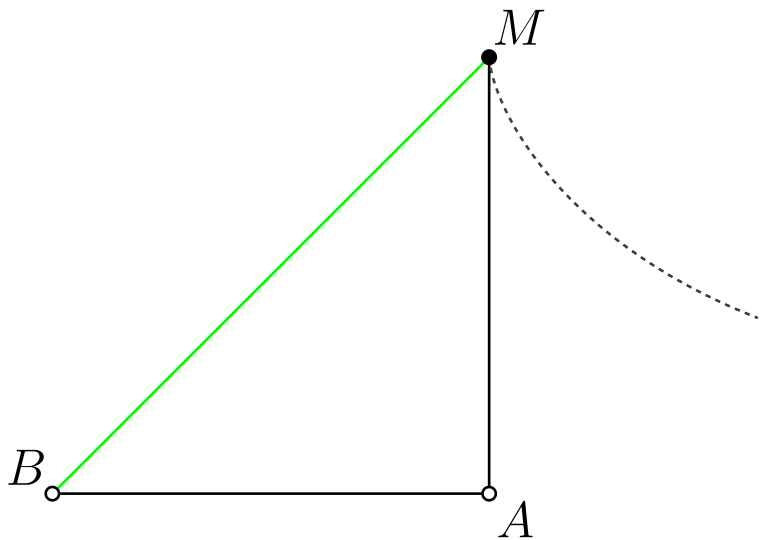

C
B
M
A

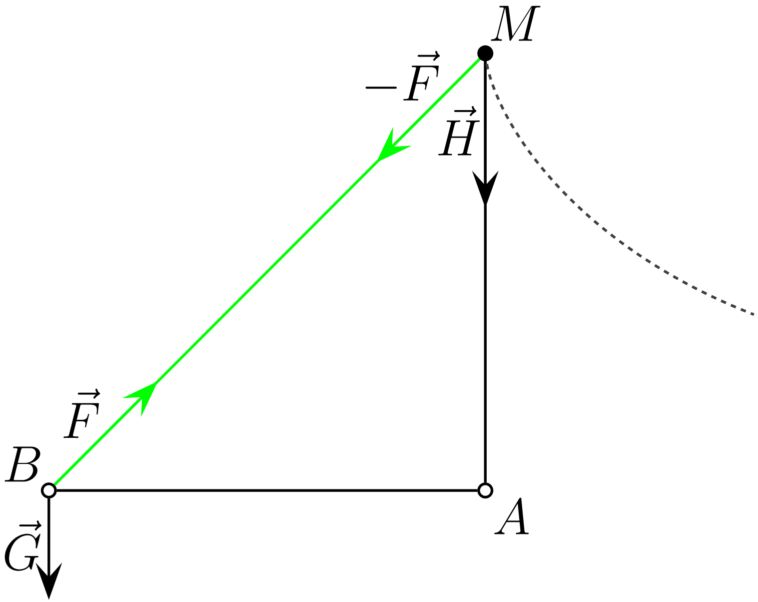

$B \equiv C$
M
A



C
M
B
A

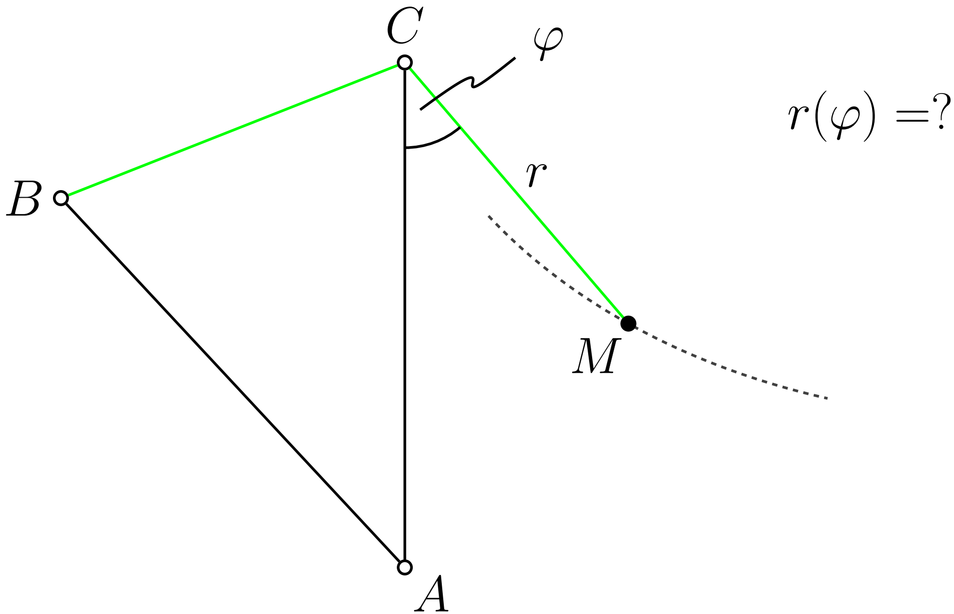

C
M
B
A

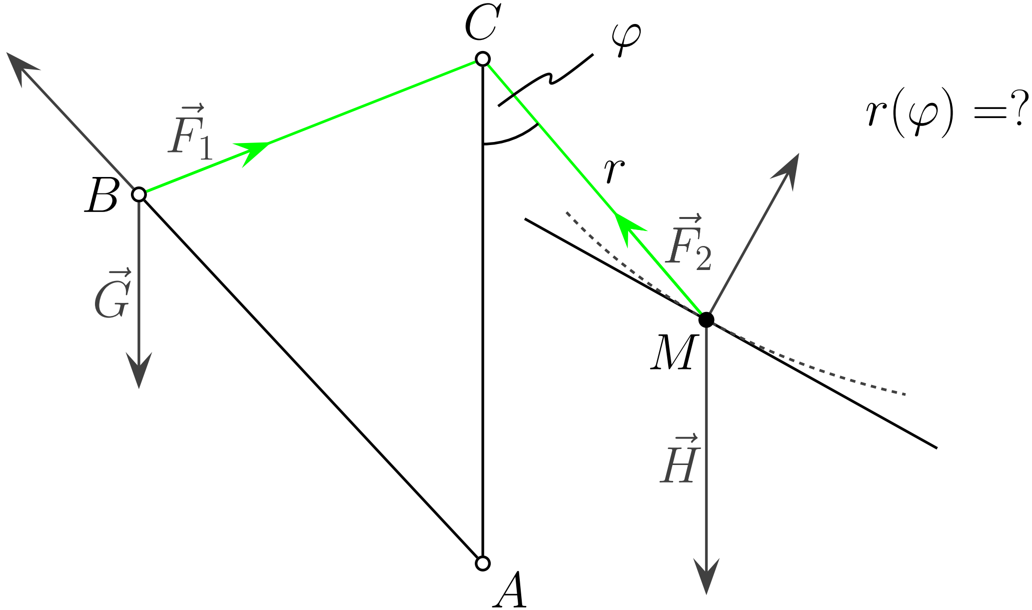

M
B
A



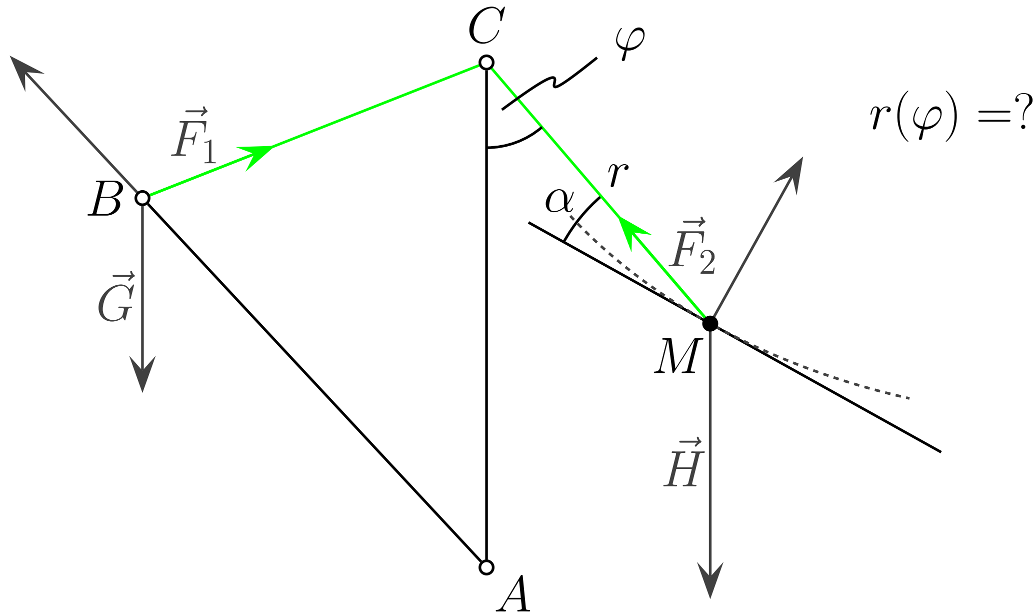
$$F = H$$

$$H = \sqrt{2}G$$

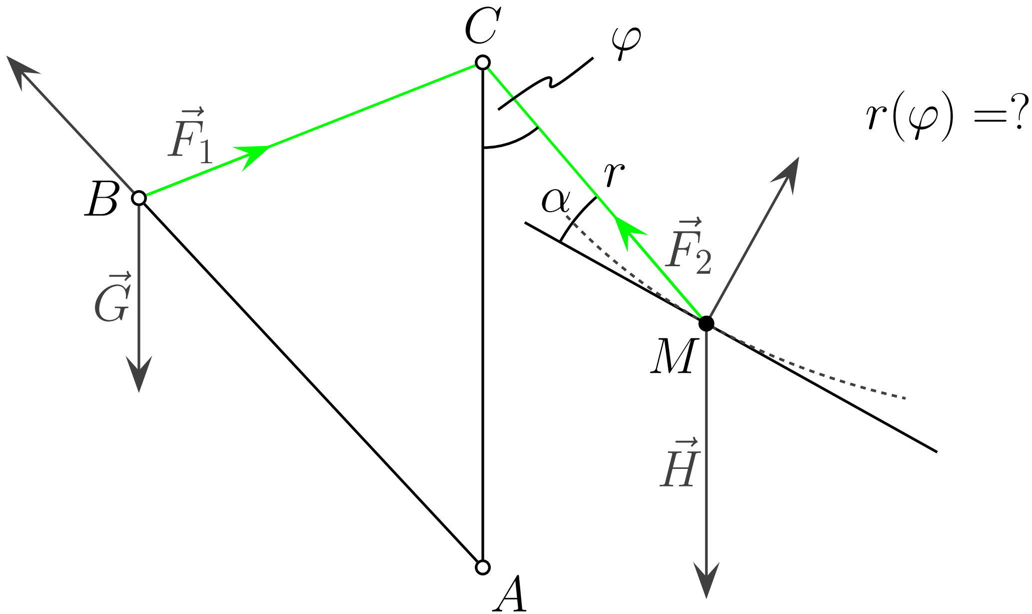

C
φ
B
r
M
A
r(φ) =?



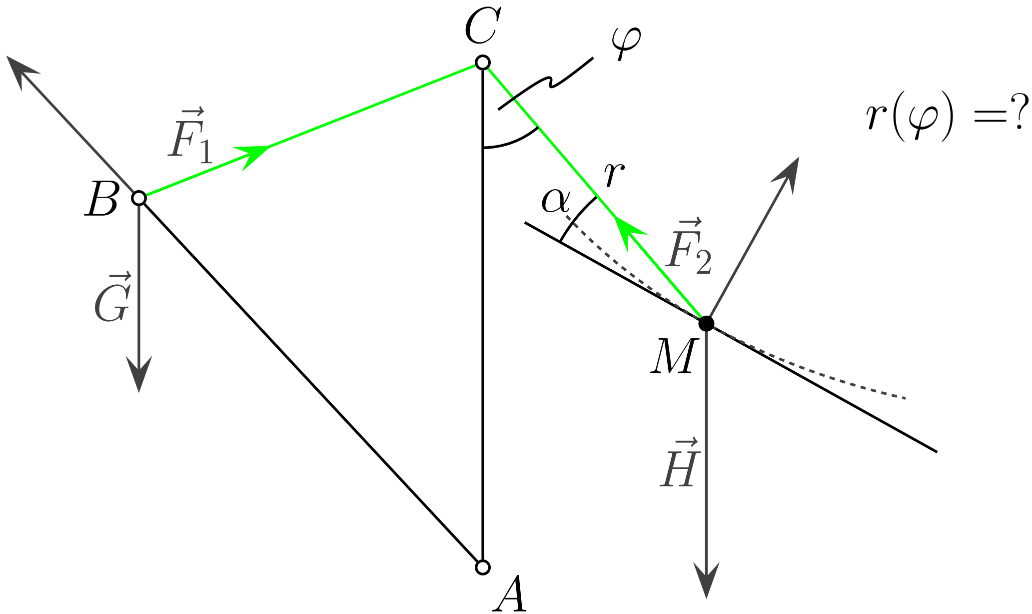
C
φ
r(φ) =?
$\vec{F}_1$
r
B
$\vec{F}_2$
$\vec{G}$
M
$\vec{H}$
A



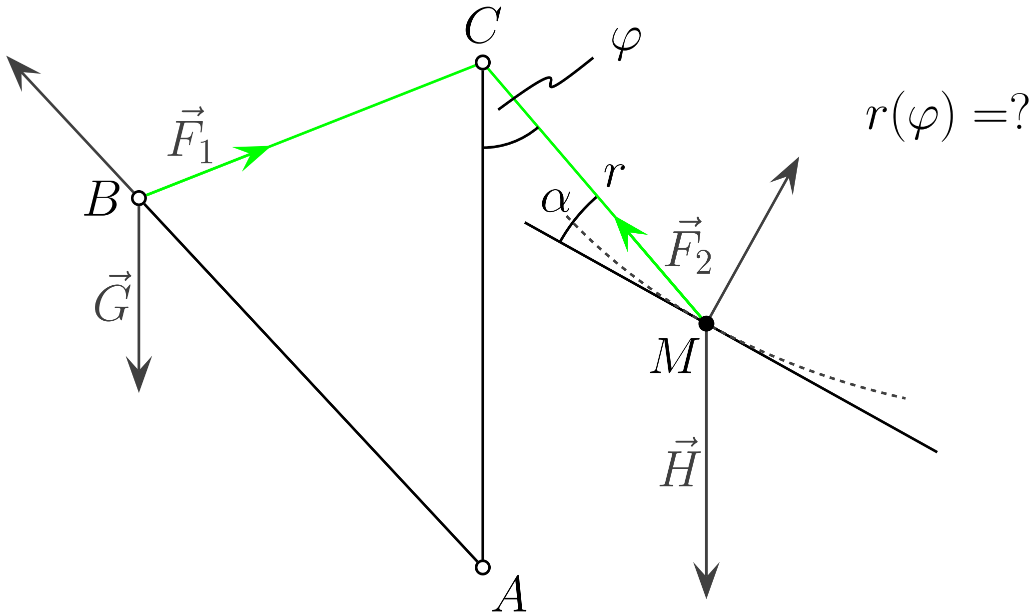
C
φ
$r(\varphi) = ?$
$\vec{F}_1$
B
r
α
$\vec{F}_2$
$\vec{G}$
M
$\vec{H}$
A



$$\cos\varphi - \sin\varphi \tan\alpha = \frac{L - r}{L}$$

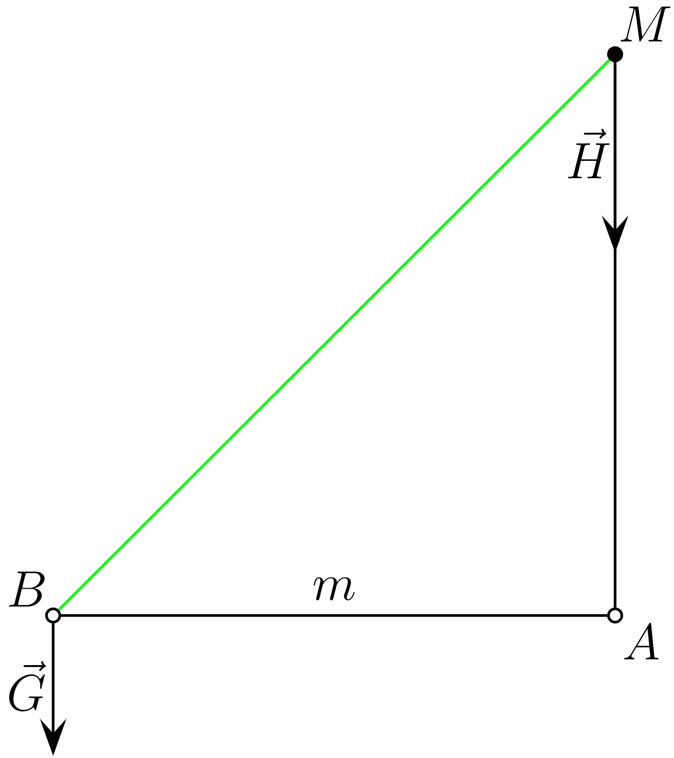


$$\cos\varphi - \sin\varphi \tan\alpha = \frac{L-r}{L}\,, \qquad \tan(\alpha) = \frac{r(\varphi)}{r'(\varphi)}$$



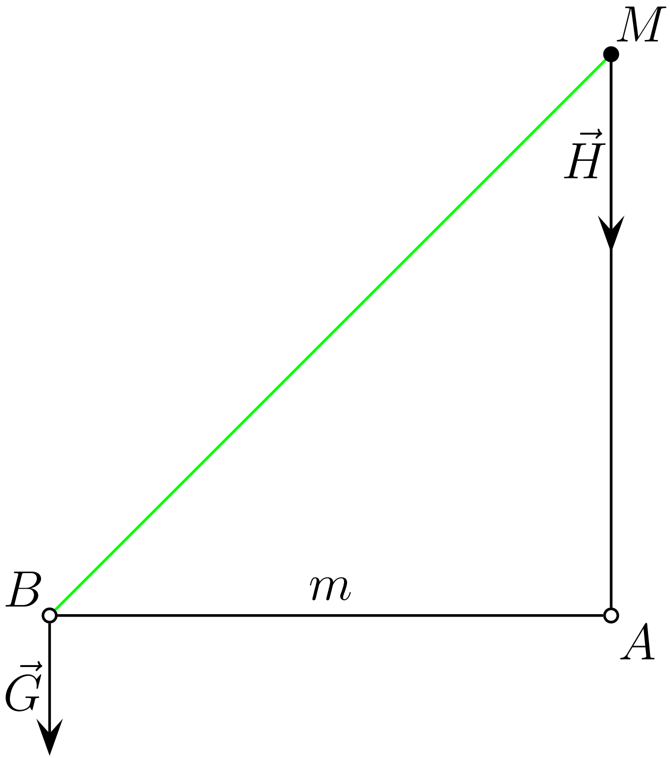
$$\cos\varphi - \sin\varphi \tan\alpha = \frac{L-r}{L}\,, \qquad \tan(\alpha) = \frac{r(\varphi)}{r'(\varphi)}$$

$$r\cos\varphi - r'\sin\varphi = \frac{r'(L-r)}{L}, \qquad r(0) = 0$$



$$H = \sqrt{2}G$$

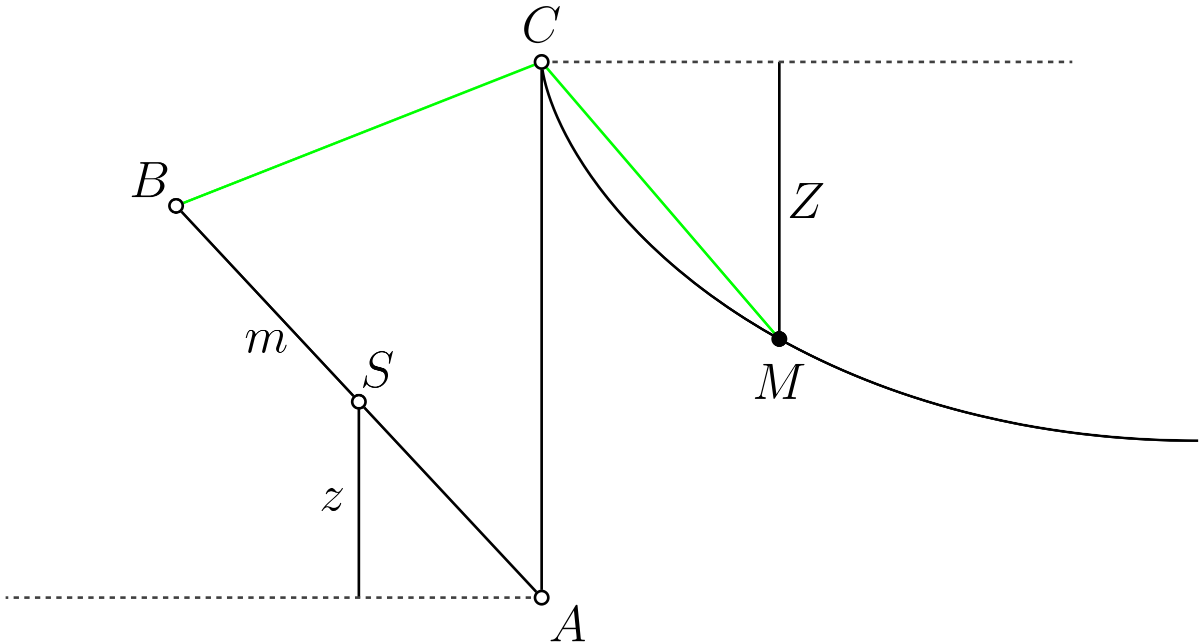
$$H = Mg, \quad G = \frac{mg}{2}$$



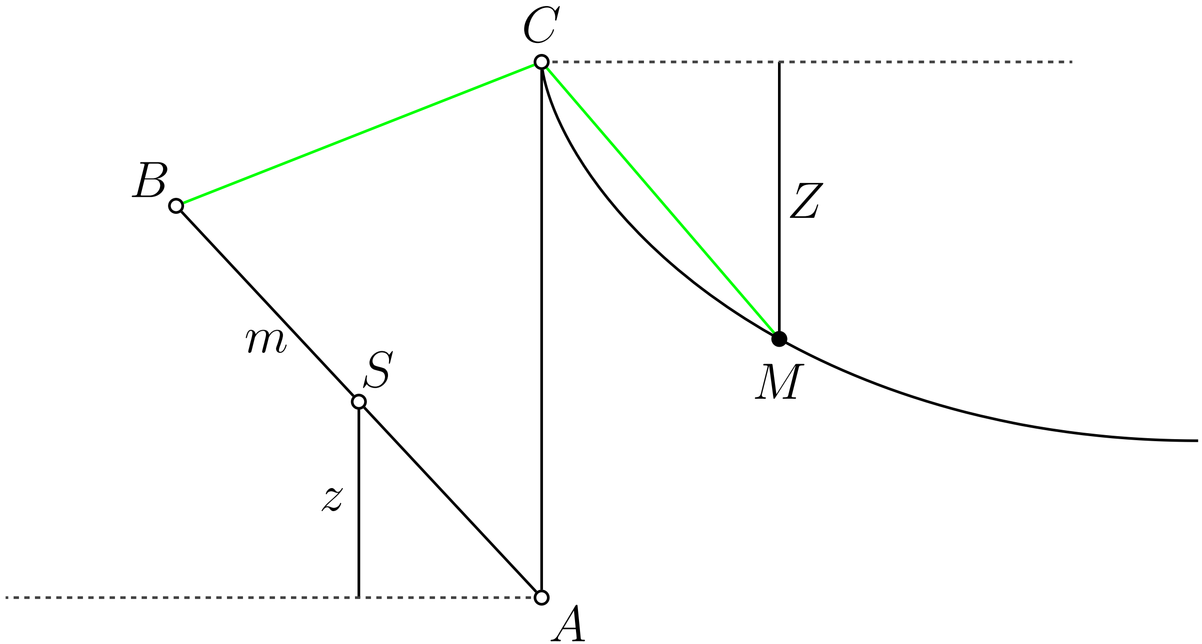
$$H = \sqrt{2}G$$

$$H = Mg, \quad G = \frac{mg}{2}$$

$$m = \sqrt{2}M$$

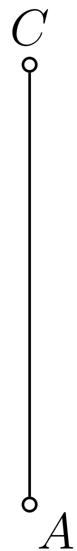


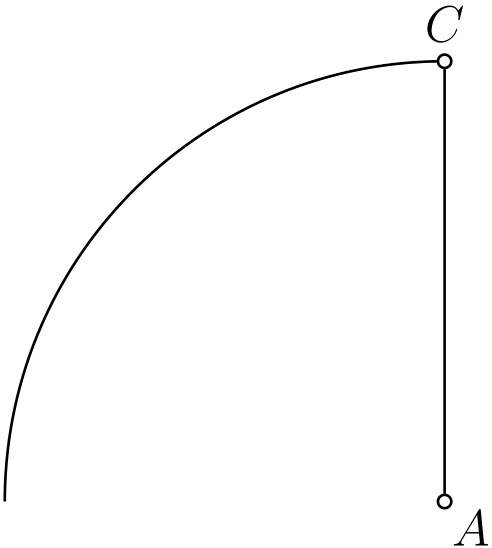
$$mgz \;=\; MgZ$$

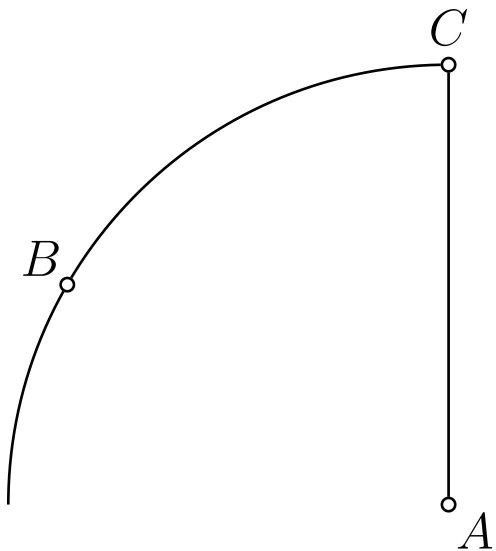


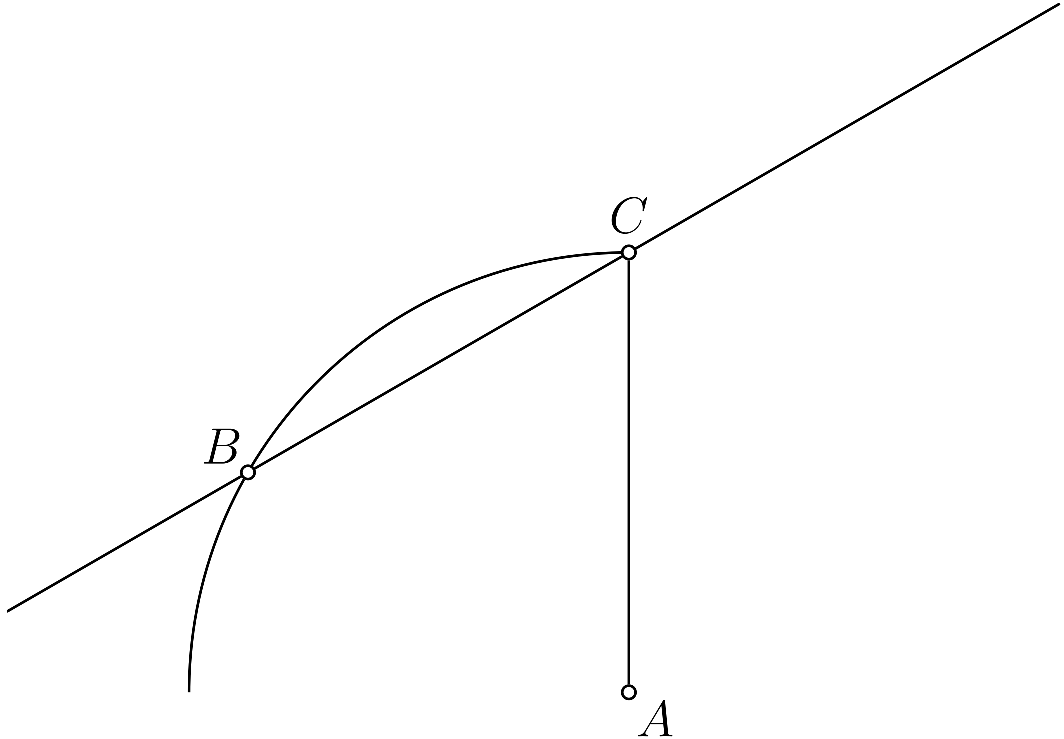
$$mgz \;=\; MgZ, \quad m = \sqrt{2}M$$

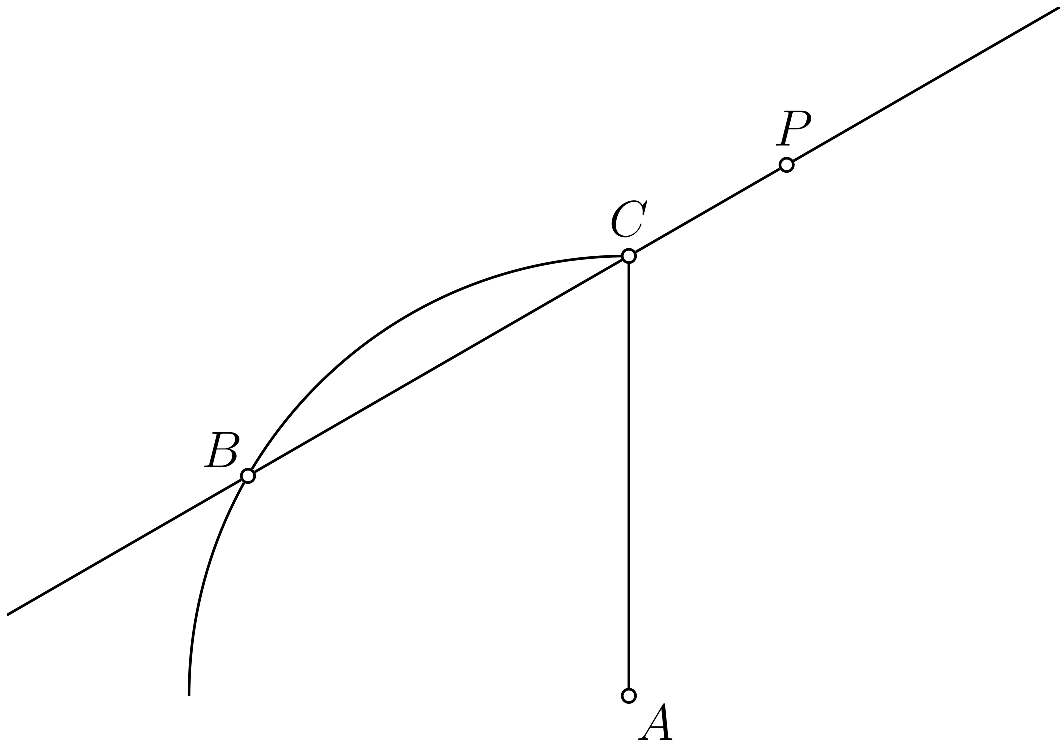
$$\iff \; Z = \sqrt{2}z$$


C
A

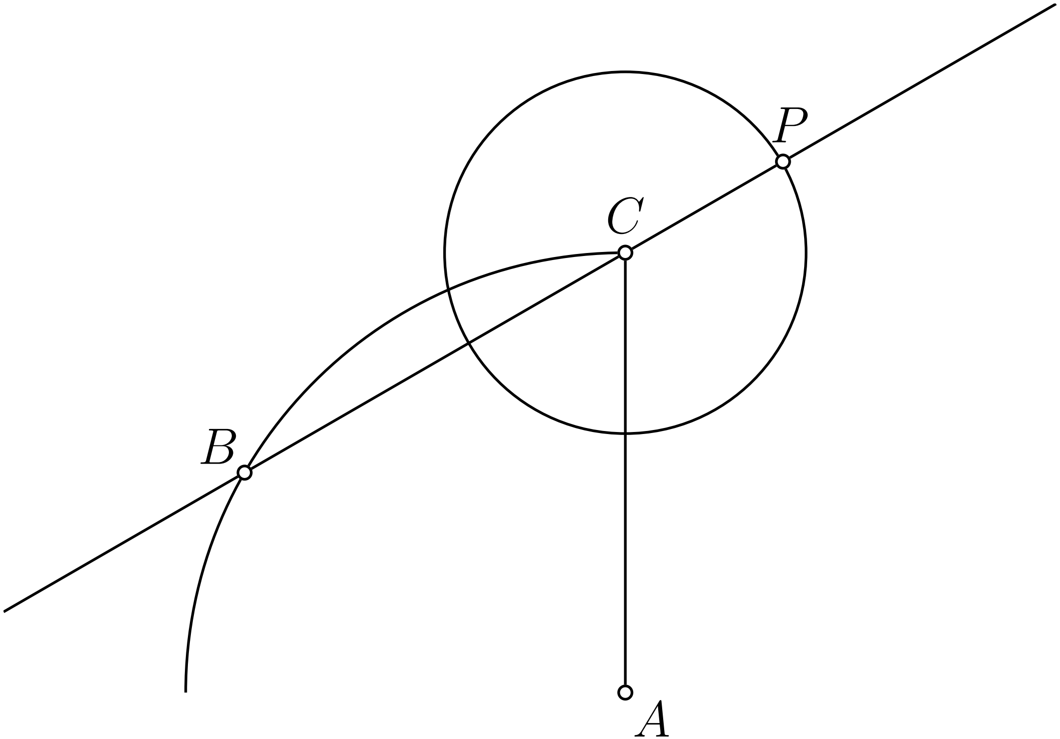

C
A


C
B
A


C
B
A

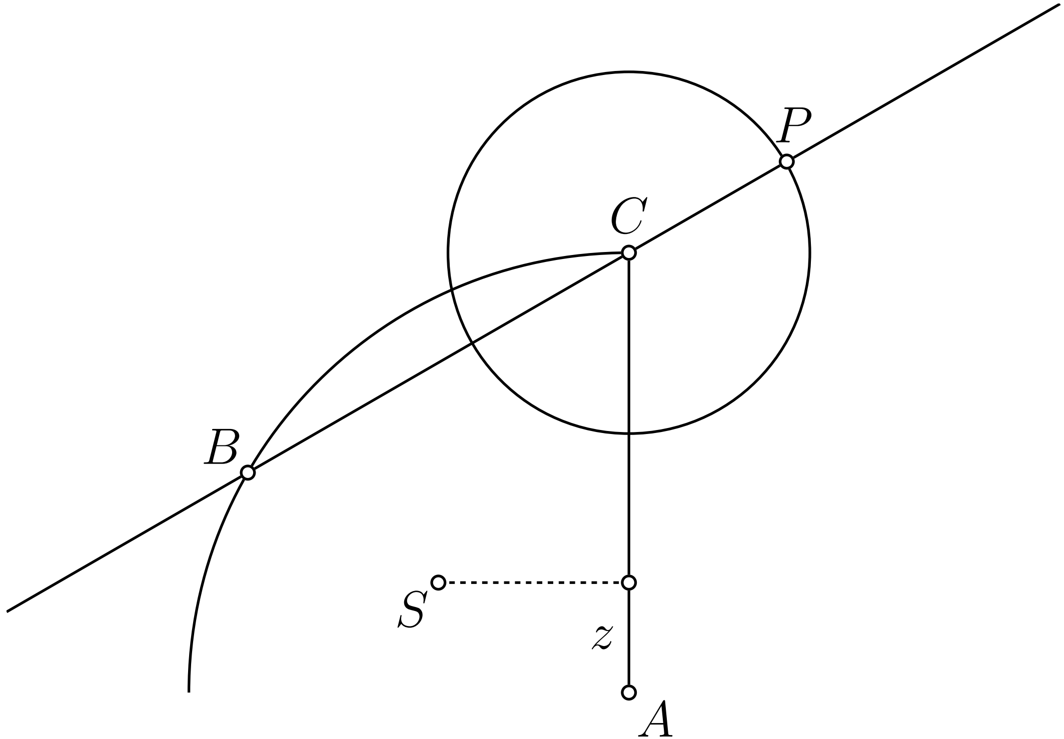


$\overline{BP} = L$



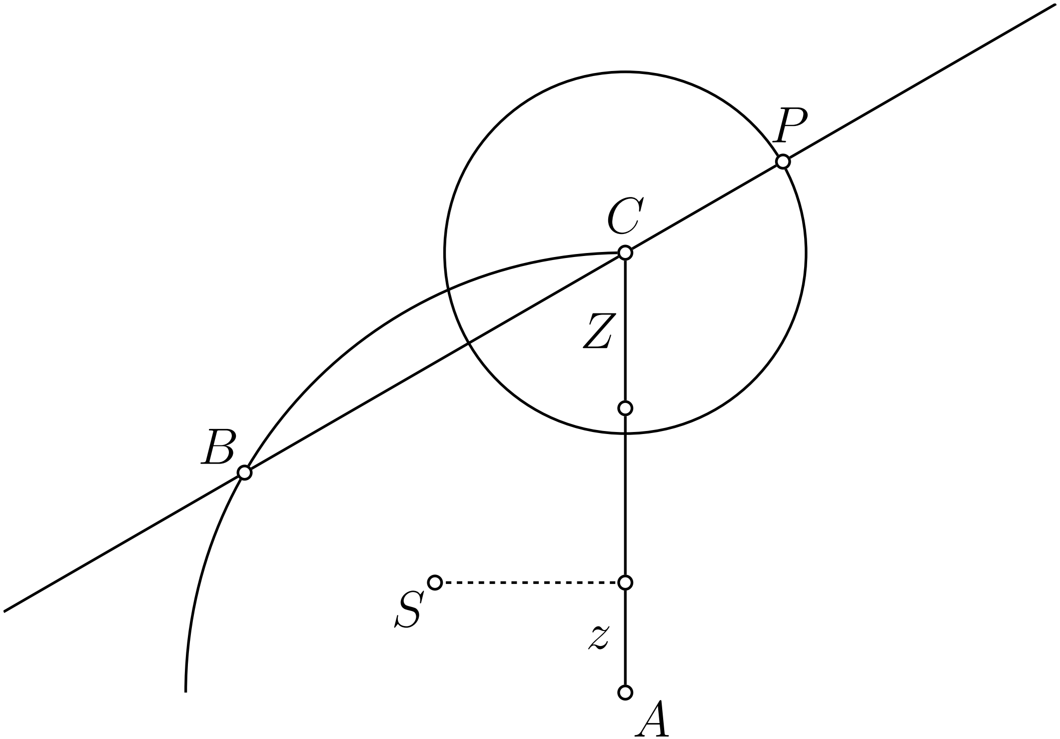
$\overline{BP} = L$

1.gO



$\overline{BP} = L$

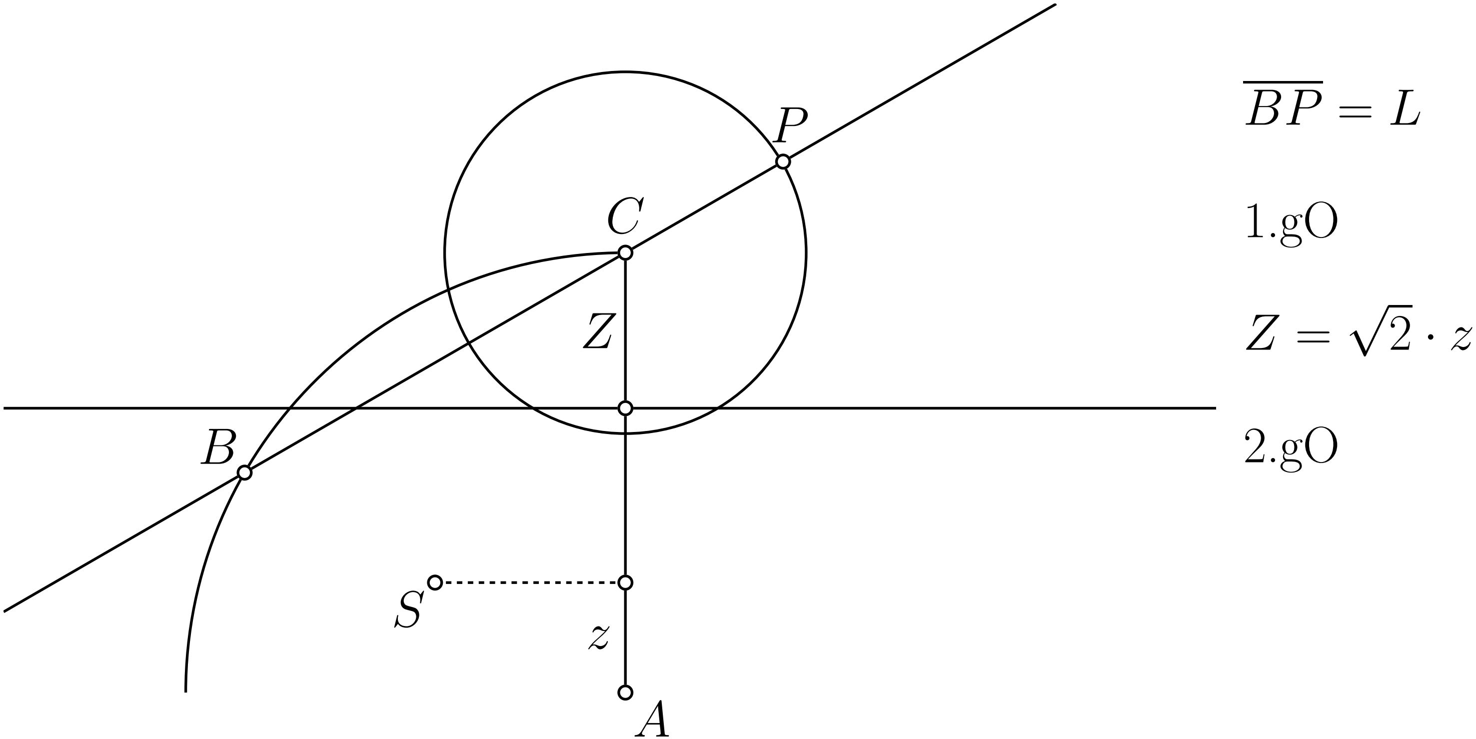
1.gO

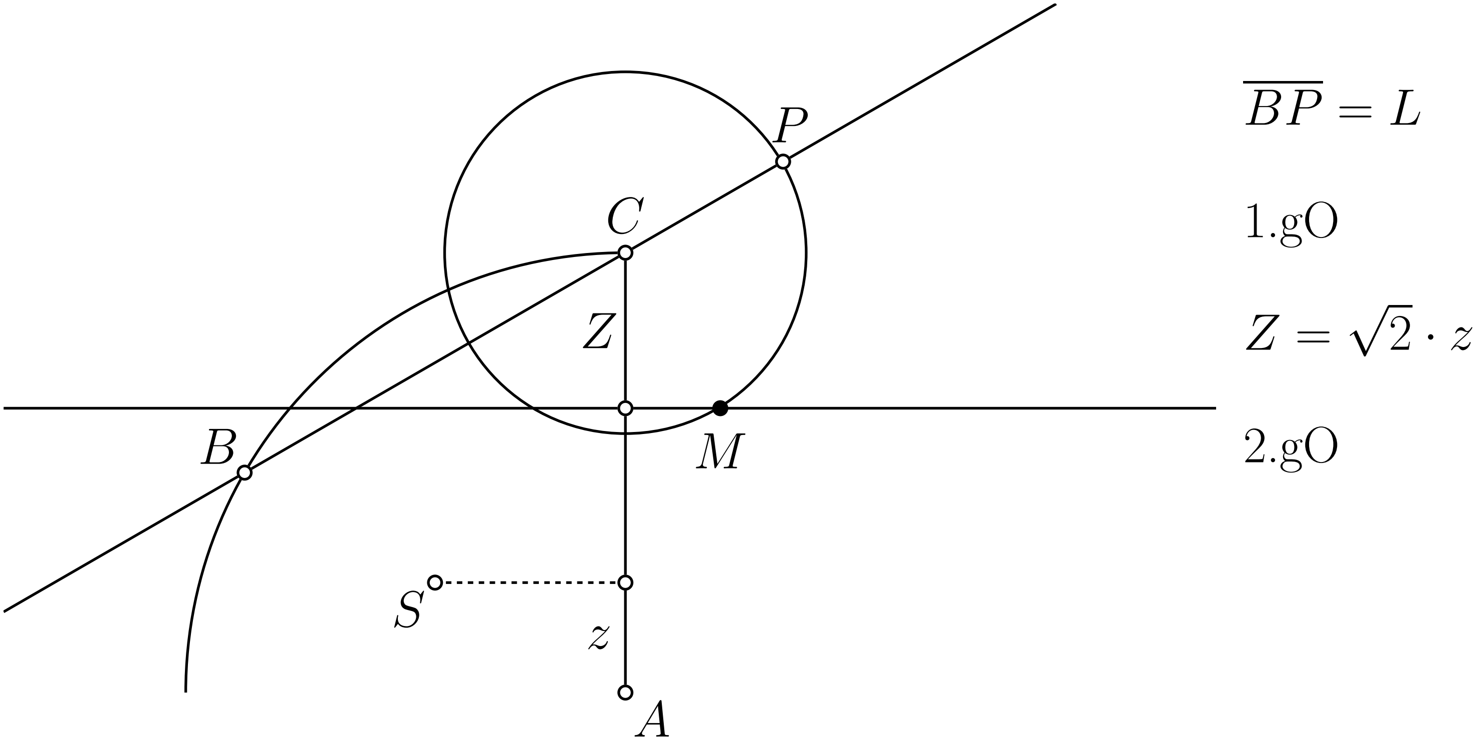


$\overline{BP} = L$

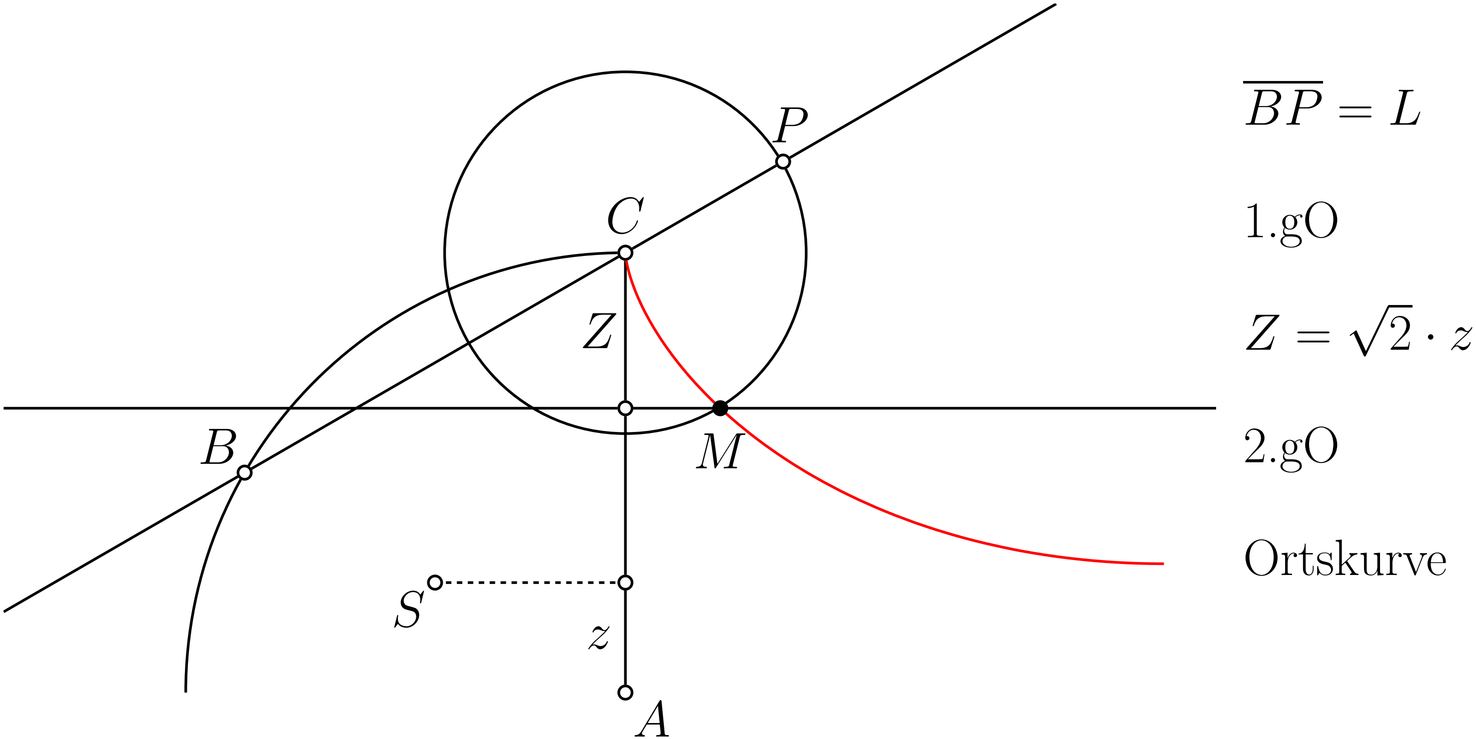
1.gO

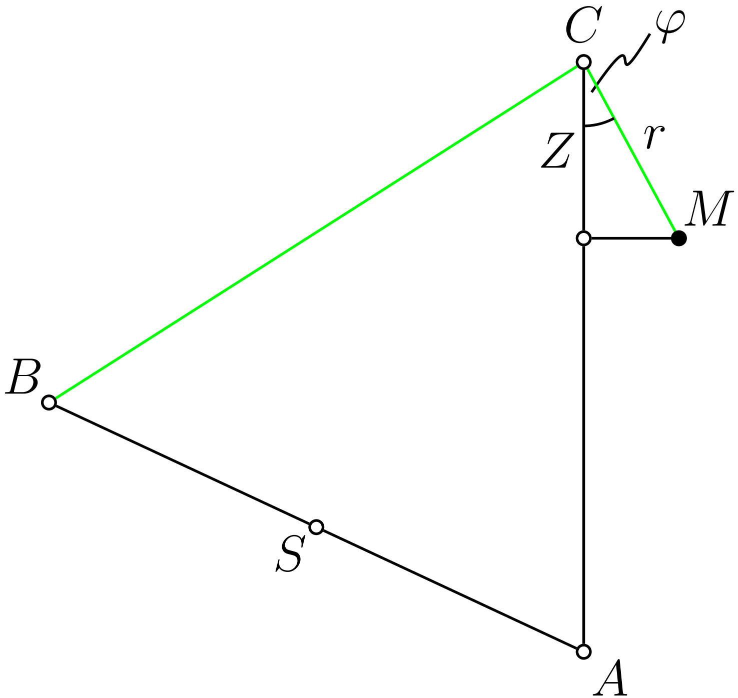
$Z = \sqrt{2} \cdot z$


$\overline{BP} = L$
1.gO
$Z = \sqrt{2} \cdot z$
2.gO
P
C
Z
B
S
z
A

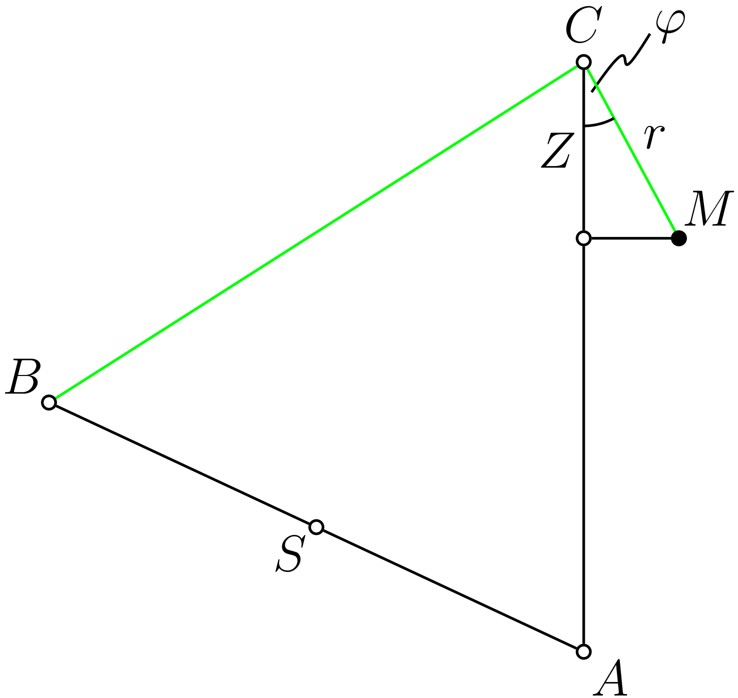


$\overline{BP} = L$
1.gO
$Z = \sqrt{2} \cdot z$
2.gO
P
C
Z
B
M
S
z
A


$\overline{BP} = L$
1.gO
$Z = \sqrt{2} \cdot z$
2.gO
Ortskurve
P
C
Z
B
M
S
z
A



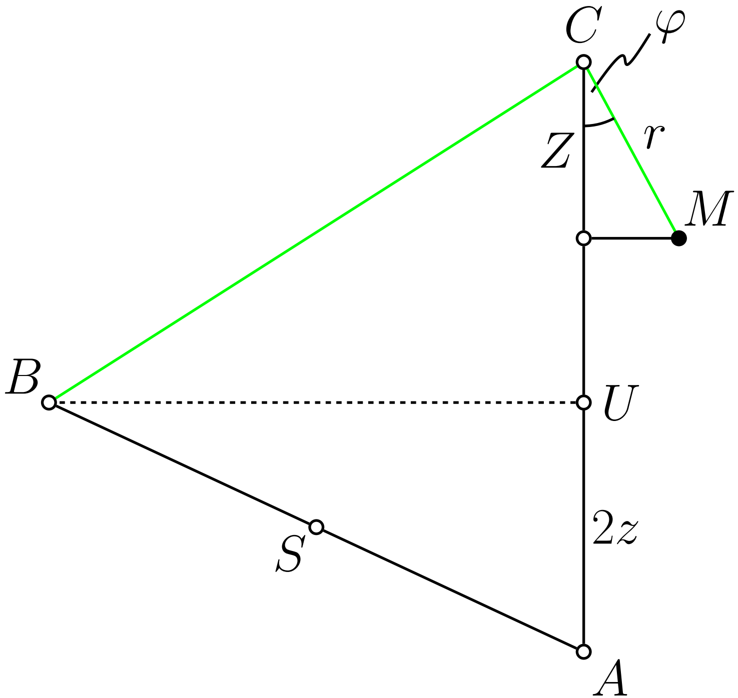
C
φ
Z
r
M
B
S
A



$$\overline{BC} = L - r$$

$$\overline{AC} = \overline{AB} = L/\sqrt{2}$$

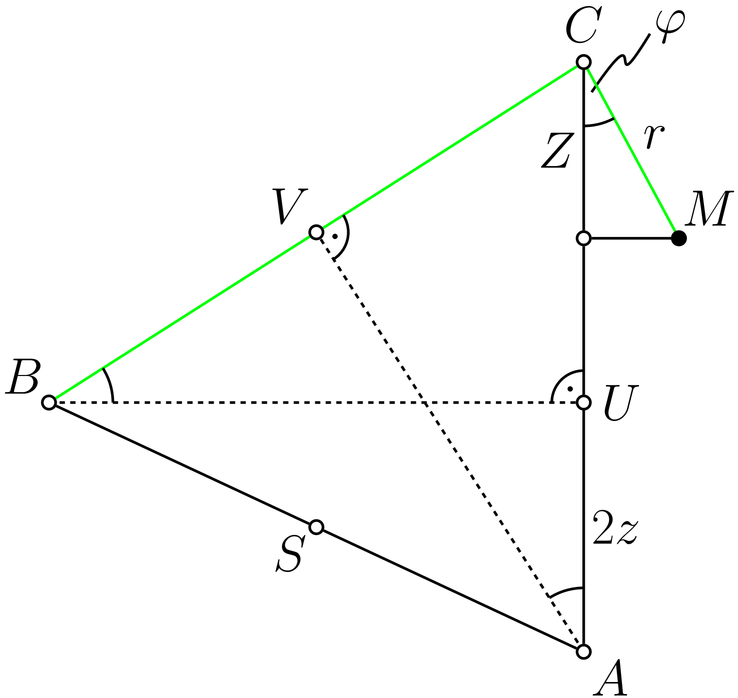
$$Z = r \cdot \cos(\varphi)$$



$$\overline{BC} = L - r$$

$$\overline{AC} = \overline{AB} = L/\sqrt{2}$$

$$Z = r \cdot \cos(\varphi), \quad Z = \sqrt{2}z$$

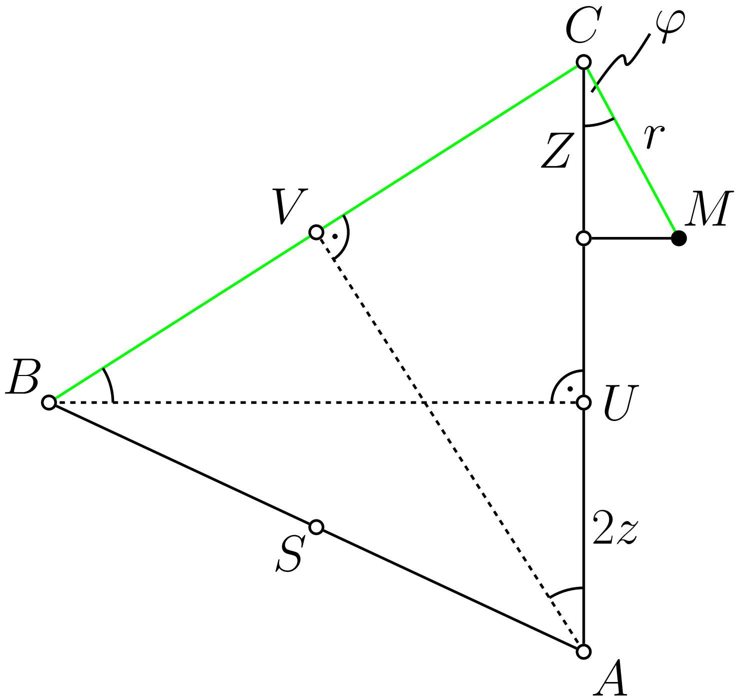


$$\overline{BC} = L - r$$

$$\overline{AC} = \overline{AB} = L/\sqrt{2}$$

$$Z = r \cdot \cos(\varphi), \quad Z = \sqrt{2}z$$

$$\Delta BUC \sim \Delta AVC$$



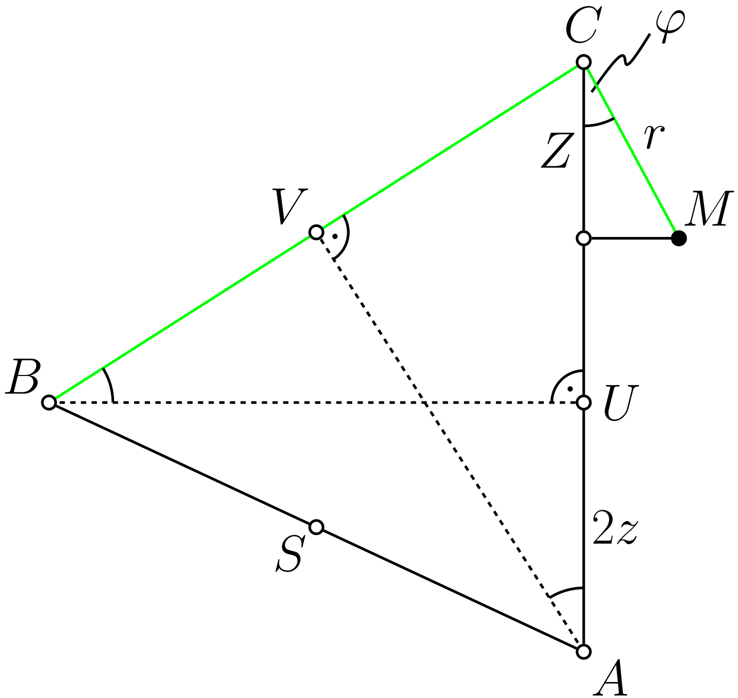
$$\overline{BC} = L - r$$

$$\overline{AC} = \overline{AB} = L/\sqrt{2}$$

$$Z = r \cdot \cos(\varphi), \quad Z = \sqrt{2}z$$

$$\Delta BUC \sim \Delta AVC$$

$$z = \frac{r(2L - r)}{2\sqrt{2}L}$$



$$\overline{BC} = L - r$$

$$\overline{AC} = \overline{AB} = L/\sqrt{2}$$

$$Z = r \cdot \cos(\varphi), \quad Z = \sqrt{2}z$$

$$\Delta BUC \sim \Delta AVC$$

$$z = \frac{r(2L - r)}{2\sqrt{2}L}$$

$$r(\varphi) = 2L(1 - \cos(\varphi))$$

$$r\cos\varphi - r'\sin\varphi = \frac{r'(L-r)}{L}, \qquad r(0) = 0$$

$$r\cos\varphi - r'\sin\varphi = \frac{r'(L-r)}{L}, \qquad r(0) = 0$$

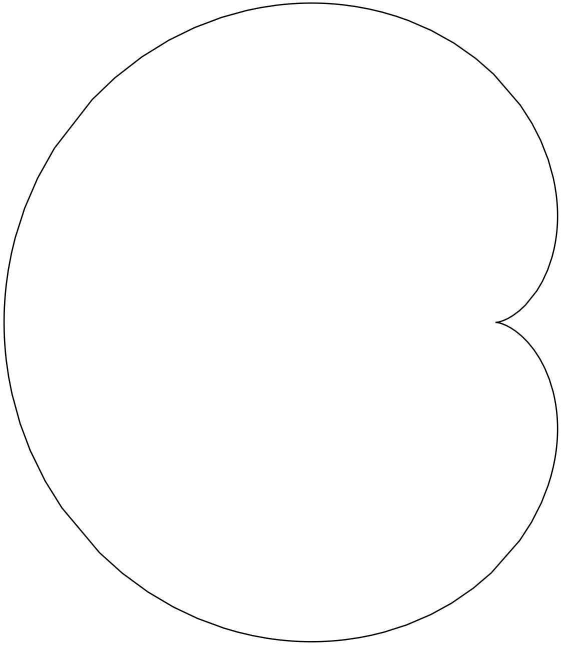
$$r(\varphi) = 2L(1-\cos(\varphi))$$

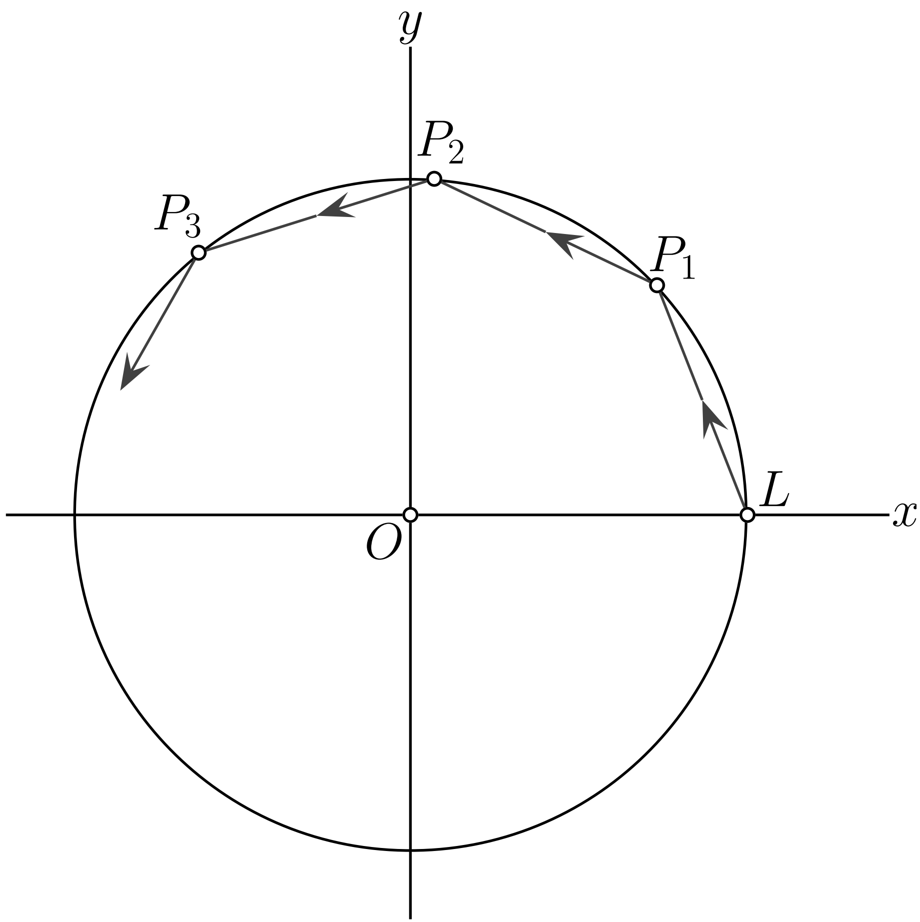
$$r\cos\varphi - r'\sin\varphi = \frac{r'(L-r)}{L}, \qquad r(0) = 0$$

$$r(\varphi) = 2L(1-\cos(\varphi))$$

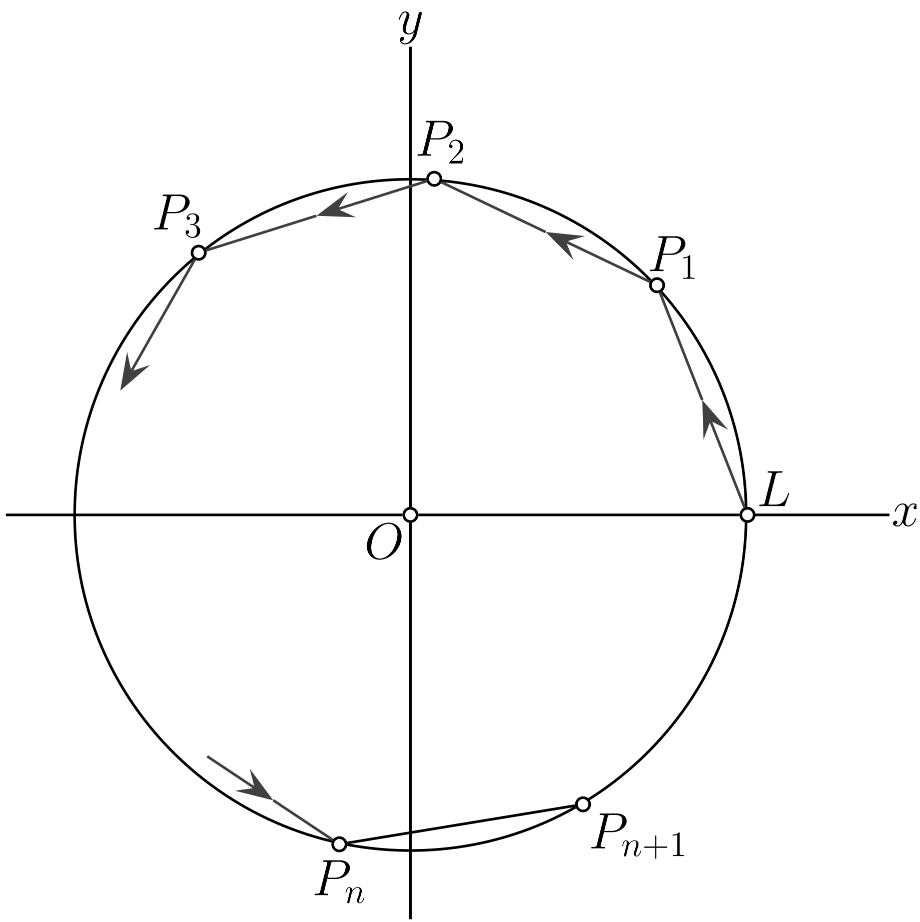
einsetzen... $\surd$

```
r[φ_] = 2 L (1 - Cos[φ]) /. L → 1;

ParametricPlot[r[φ] {Cos[φ], Sin[φ]}, {φ, 0, 2 Pi},
              AspectRatio → Automatic, Axes → False]
```

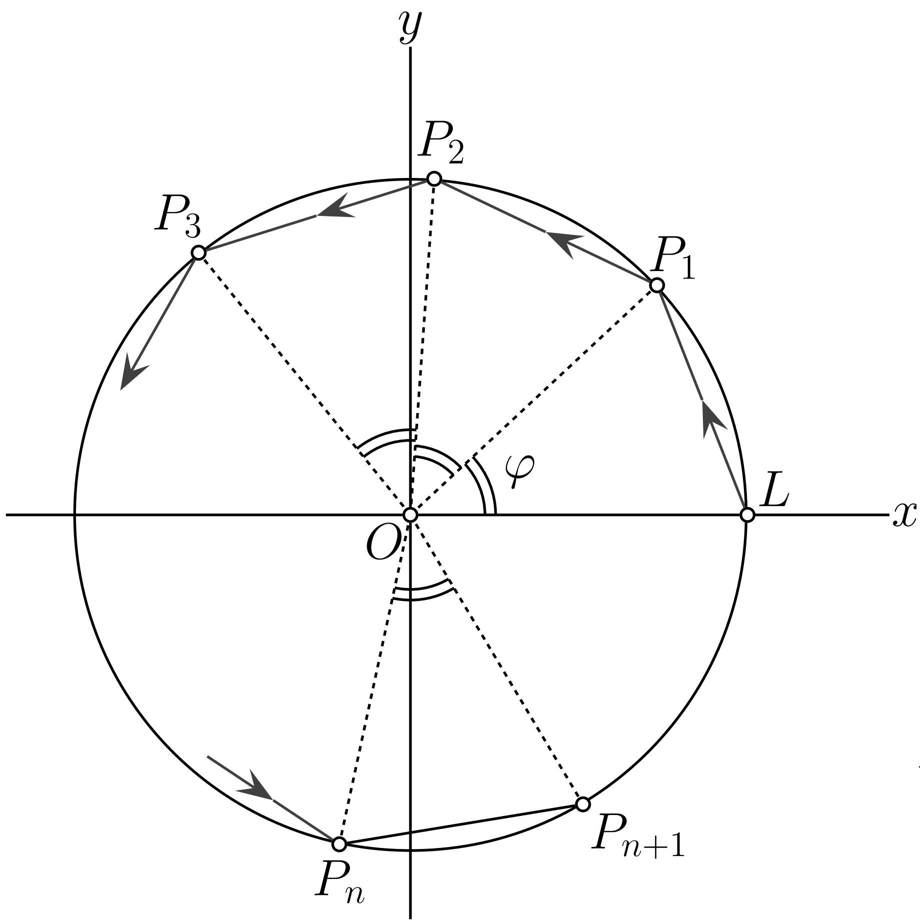




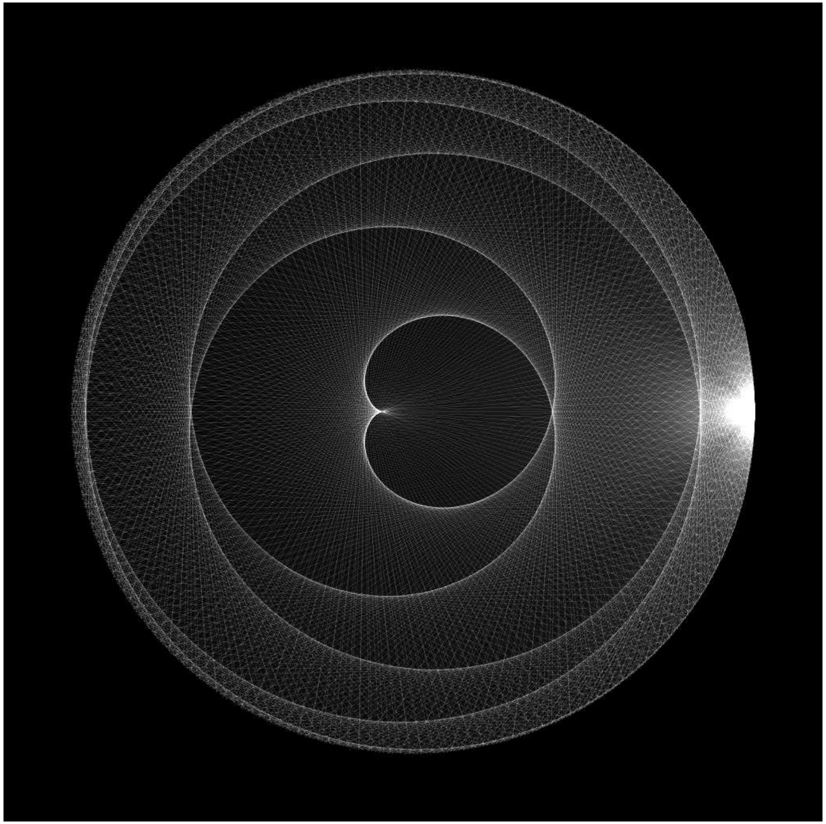
y
P_2
P_3
P_1
L
x
O



y
P2
P3
P1
L
x
O
Pn
Pn+1



$$P_n(\cos(n\varphi), \sin(n\varphi))$$



```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_, n_] = {Cos[n φ], Sin[n φ], 1};
g[φ_, n_] = Verb[p[φ, n], p[φ, n + 1]];
```

```
Schneid[u_, v_] := Cross[u, v];
Verb[u_, v_] := Schneid[u, v];

Huellkurve[u_, t_] := Cross[u, D[u, t]];
Ortskurve[u_, t_] := Huellkurve[u, t];

Cart[u_] := Delete[u / u[[3]], 3]

p[φ_, n_] = {Cos[n φ], Sin[n φ], 1};
g[φ_, n_] = Verb[p[φ, n], p[φ, n + 1]];

k[φ_, n_] = Huellkurve[g[φ, n], φ] // Cart // FullSimplify
```

$$\left\{\frac{(1+n)\,\mathrm{Cos}[n\,\varphi]+n\,\mathrm{Cos}[(1+n)\,\varphi]}{1+2\,n},\ \frac{(1+n)\,\mathrm{Sin}[n\,\varphi]+n\,\mathrm{Sin}[(1+n)\,\varphi]}{1+2\,n}\right\}$$

```
ParametricPlot[k[φ, 5], {φ, 0, 2 Pi},
        AspectRatio → Automatic, Axes → False, PlotPoints → 80];
```